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Review Article

Impacts of Exponentially Growing/Decaying Pressure Gradient on Mixed Convection Flow of Viscous Reactive Fluid in a Vertical Tube: A Numerical Approach

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Abstract

This study examined the effects of a pressure gradient that was exponentially increasing or decreasing on the mixed convection flow of viscous reactive fluids in a vertical tube between two concentric tubes with r = 0 and r = b. The nonlinear partial differential equation governing the flow formation are solved using the implicit finite difference form where all time differentials were calculated using the forward difference formula, and second order central differences were utilized to approximate the first and second derivatives. The impacts of several physical parameters, including shear stress, the rate of heat transfer, mixed convection, viscous reactive fluid parameter, activation energy, Prandtl number, and exponential decaying/growing pressure gradient on skin friction and Nusselt number were investigated. It's intriguing to see that raising the values of Frank–Kamenetskii (λ), mixed convection (Gre) as well as exponential growing pressure term increases the velocity fluid, However, in the case of exponential decay, the parameter drop also causes the fluid's velocity to diminish. The results also showed that a small bump in the viscous reactive fluid parameter considerably improves the energy profile.

Keywords: Implicit finite difference scheme, Exponentially Decaying/Growing, Pressure gradient, viscous reactive fluid.

1. INTRODUCTION

Much emphasis has been paid to the investigation of mixed convective flow along a rigid plate lately because it is essential for a variety of applications, including the electronic cooling devices by fans, the cooling of nuclear reactors during an unexpected failure, the placement of heating systems in minimal-velocity environments, solar cells, and others Aydin and Kaya, [1] examined MHD mixed convection of a viscous dissipating fluid about a vertical cylinder, concluding that slight increase in mixed convection increase the velocity fluid. Hamza *et al.*, [2] inspect mixed convection flow of viscous reactive fluids with thermal diffusion and radial magnetic field in a vertical porous annulus. Recently Jafar *et al.*, [3] reported that the buoyant force has a substantial impact on the flow field for vertical or inclined surfaces. Mixed convection can exploit in dispersion of chemical contaminants in various processes and in the chemical industry, transport system for heated or cooled fluids and many others as reported by Lavine [4]. Malvandi *et al.*, [5] presented a theoretically analysis on MHD mixed convection of alumina/water nanofluid inside a vertical annular pipe. Hayat *et al.*, [6] investigated mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. The results of the research in [7-12] details more application of mixed convection. Mixed convection flow in a vertical tube appears in many practical engineering applications, including heat exchangers, chemical processing equipment, geothermal energy extraction, food processing, casting and welding of a manufacturing process, the dispersion of chemical contaminants in various processes and in the chemical industry, transport system for heated or cooled fluids. Hamza and Abdulsalam [13] investigate the Influence of chemical kinetic exponent on transient mixed convective hydromagnetic flow in a vertical channel with convective boundary condition. Halim and Noor [14] Scrutinized Mixed convection flow of nanofluid near stagnation point along a vertical stretching. Gul *et al.*, [15] Proposed heat transfer in MHD mixed convection flow of a ferrofluid along a vertical channel.

The study of fluid dynamics that embedded through curved geometry nowadays has received a great interest and concern due to its applications in science and engineering. This can be ascribed to its practical use in most of the systems in the aforementioned fields of endeavor. For instance, in diaphragm pumps and dialysis machines, flow is due to pulsation within the curved tube rather than convective current. It is widely known that the underlying principle of convective-driven flows has its setbacks as heat alters the rheological properties of some fluids. Consequently, it will be desirable to model and design pressure-propelled systems. Dean [16] was the first researcher to initiated flow in a curved channel with azimuthal pressure gradient. Afterwards, a considerable number of works have dealt with investigations of the effects of slip boundaries on Dean flows for various flow conditions [17–24]. Jha and Gambo [25] reported in their findings that maximum Dean velocity is due to an exponentially growing time-dependent pressure gradient and slip wall coefficient. Stability of the Dean vortices is achieved by suppressing time, wall slippage and inducing an exponentially decaying time-dependent pressure gradient. Reference [26] numerically examined the impact of aspect ratio on Dean hydrodynamics instability. Gupta *et al.*, [27] simulated the flow of cerebrospinal fluid in the human spinal cavity by presenting analytical solution responsible for pulsatile viscous flow driven by a harmonically oscillating pressure gradient in a straight elliptic annulus.

It is important to notice that exponential pressure gradient plays a vital role in flow through channels as well as annulus. Such flow aids in better understanding many technological and industrial problems. In general, this phenomenon generates pressure gradient flow which is not constant but pulsates in some way about a nonzero pressure gradient. In view of that, Yen and Chang [18] examined the effect of time-dependent pressure gradient on magnetohydrodynamic flow in a channel in which three cases of time-dependent pressure gradient were considered, namely periodic, step and pulse pressure gradient. Numerical modelling of unsteady flow with time-dependent pressure gradient using one-dimensional Navier–Stokes Equation (NSE) was reported by Azad and Andallah [19]. Other related articles can be seen in references [20–23]. In slip flow regime, the heat and mass transfer effect of magnetohydrodynamic fluid flow with slip velocity was scrutinized by Gambo and Gambo [24]. Jha and Yahaya [25] examined the impact of wall slippage in a curvilinear annulus induced by azimuthal pressure gradient. The instability of Dean flow and slip condition in a rotating cylinder was analysed by Avramenko *et al.*, [26]. Authors of reference [27] explored the effect of partial slippage on a rotating flow of hydromagnetic micropolar nanoparticles flowing through a disk. These authors reported that the classical effect of slip jump is in achieving a more amplified velocity profile.

The aim of this work is to present the impact of exponentially growing/decaying pressure gradient on mixed convection flow of viscous reactive fluids in a concentric tube. The governing equations along with the initial and boundary conditions are solved by implicit finite difference scheme approach. The solutions for velocity and temperature were analyzed through graphical. However, skin friction and Nusselt number were analyzed in tabular. The applications of these fluid properties as they affect the thermodynamics, rate of heat transfer and shear stress between the concentric tube and the fluids could bound in lubrication industries, food processing and food preserving industries, cooling of electric appliances, drilling of petroleum products, etc.

2. Geometry definition and boundary conditions

We consider time dependent fully developed Laminar flow of an incompressible viscous fluid passing through concentric tube (see the Figure 1). The $z' \rightarrow$ axis is taken along the axis of the tube. The $r\rightarrow$ axis (radial co-ordinate) is taken normal to the tube. Exponentially growing/decaying pressure gradient is introduced which set up mixed convection currents inside the tube.

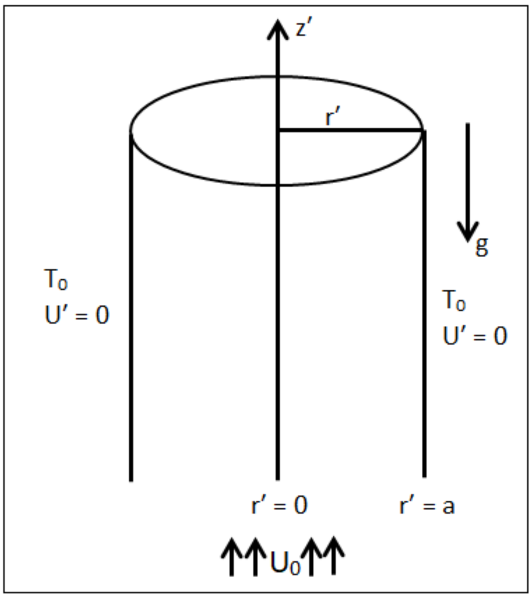

Fig 1: Geometrical illustration for the consider model

Assumptions of the present problem

1. The $Z' \rightarrow$ axis is taken along the axis of the tube.
2. The $r\rightarrow$ axis (radial co-ordinate) is taken normal to the tube.

3. The fluid is assumed to be at the same temperature (T_0) and there is no fluid motion at $t' \leq 0$.
4. At $t' > 0$ fluids start exothermic heat generation due to its reactive nature.
5. Exponentially growing/decaying pressure gradient is introduced which set up mixed convection currents inside the tube.
6. The fluid is assumed to be fully developed has constant physical properties and obeys the Boussineqs approximation.

The governing equations in Dimensional form:

$$\rho\frac{\partial u'}{\partial t'}=-\frac{\exp(-\delta_0 t')}{r'}\frac{\partial P}{\partial \varphi}+\frac{v'}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u'}{\partial r'}\right)+g\beta\left(T'-T_0\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\partial T'}{\partial t'}=\frac{k}{\rho C_p}\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'}{\partial r'}\right)+\frac{QC_0A}{pc\rho}\exp\left(-\frac{E}{RT'}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Boundary Conditions for the problem,

$$t\leq 0:\quad u'=0,\ T'=T_0 \quad for\ 0\leq r'\leq a$$

$$t'>0:\frac{\partial u'}{\partial r'}=0,\ \frac{\partial T'}{\partial r'}=0 \quad at\ r'=0 \quad \ldots\ldots\ldots\ldots (3)$$

$$u'=0,\ T'=T_0 \quad at\ r'=a$$

The non-dimensional quantity is given by

$$\left.\begin{aligned} & r=\frac{r'}{a},\ t=\frac{vt'}{a^2},\ U=\frac{u'}{U_0},\theta=\frac{E}{RT_0^2}[T'-T_0],\ \mathrm{Re}=\frac{U_0 a}{v} \\ & U_0=\frac{-r\frac{\partial p}{\partial \varphi}\exp(-\delta_0 t')}{pU'},\ \delta=\frac{r^2\delta_0}{v},Gr=\frac{g\beta RT_0^2 r^3}{Ev}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\frac{\partial u}{\partial t}=\frac{e^{-\delta t}}{r}+\frac{Gr}{\mathrm{Re}}\theta+\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\Pr\frac{\partial \theta}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \theta}{\partial r}\right]+\lambda\exp\left(\frac{\theta}{1+E\theta}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The relevant boundary Condition for all the problems are:

$$\frac{\partial u}{\partial r}=0,\quad \frac{\partial \theta}{\partial r}=0 \quad at\ r=0$$

$$u=0,\quad \theta=0 \quad at\ r=1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

3. NUMERICAL SOLUTION

The IFD method is used to solve the pair of nonlinear systems (1) and (2) with setting conditions. For all time differentials, we used the forward difference formula and approximated first and second derivatives with second order central differences. The equations reflect the initial components, and the final nodes are changed to account for the model parameters. For settings (5) and (6), the IFD equation is as follows:

$$-R_1U_{i-1}^{j+1}+(1+2R_1)U_i^{j+1}-R_1U_{i+1}^{j+1}=U_i^j+R_2\left[U_{i+1}^j-U_{i-1}^j\right]+R_3+Gre\Delta t\theta_i^j \quad \ldots\ldots\ldots\ldots\ldots (8)$$

$$-R_1\theta_{i-1}^{j+1}+(\Pr+2R_1)\theta_i^{j+1}-R_1\theta_{i+1}^{j+1}=\Pr\theta_i^j+R_2\left[\theta_{i+1}^j-\theta_{i-1}^j\right]+\Delta t\lambda\exp\left(\frac{\theta_i^j}{1+\varepsilon\theta_i^j}\right) \quad (9)$$

With boundary condition

$$\left.\begin{array}{lll}\dfrac{-U_{i-1}^{j+1}+4U_i^{j+1}-U_{i+1}^{j+1}}{2\Delta r}=0, & \dfrac{-\theta_{i-1}^{j+1}+4\theta_i^{j+1}-\theta_{i+1}^{j+1}}{2\Delta r}=0, & r=0\\ U_i^j=0, & \theta_i^j=0, & r=1\end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Where $R_1=\dfrac{\Delta t}{(\Delta r)^2},\quad R_2=\dfrac{\Delta t}{2r(i)\Delta r},\quad R_3=\dfrac{\Delta t e^{-\delta t}}{r(i)}$

4. RESULT OF THE FINDINGS

The effects of an exponentially increasing/decreasing pressure gradient on the mixed convection flow of a viscous reactive fluid in a vertical tube have been computed in a previous investigation. A MATLAB program is created to analyze and display graphs and numerical values for Dean velocity, skin frictions, in order to get insight into the physical issue. Time (t), the pressure gradient's decaying/growing parameter (δ), and the mixed convection parameter Gre are all observed to have an influence on the flow. Our current computation was done across a reasonable range of numbers and took some time. $0.1 \le t \le 0.5$, decaying/growing parameter of pressure gradient over $-2.0 \le \delta \le 2.0$. Throughout our investigation, $\delta > 0$ has been used to represent an exponentially growing pressure gradient and $\delta < 0$ is denoted as exponentially decaying pressure gradient. Figs 2, 3, 4 & 5 depicted the graphical illustration of embedding governing parameters in the fluids. However, Tables 1-5 portrayed the reaction of embedded parameter in rate of heat transfer and shear stress. Figure 2 show the effect of changing a viscous reactive fluid parameter for a specific amount of time on velocity. It was illuminating that, as stated by Hamza and Abdulsalam [14], the velocity fluid rate in the bottom channel is increasing, and energy fluids improve when the thermostat's strength and the amount of sticky heating source substances rise. Fig 3 reveal the influence of exponentially growing/decaying pressure gradient in respect with time (t), it is noted that for a pressure gradient that increases exponentially over time, the velocity profile is greater and more pronounced in (3a), whereas for a pressure gradient that decreases exponentially over time, the opposite phenomenon is noted Fig (3b). Figures 4a and 4b, respectively, show the fluctuation of mixed convection (Gre). It was shown that Gre both increases the fluid in both scenarios of exponential decaying or growing; this has a wide range of uses, including the cooling of nuclear reactors in the event of an unexpected failure and electrical cooling devices using fans.

The influence of the viscous reactive fluid parameter and Prandtl number over a time on the energy profile was shown in Figs 5a and 5b. It was found in Fig 5a that the viscous heating in the energy equation, which causes a fast increase in the temperature profile, causes the fluid profile to surge with time. The impact of the Prandtl number (Pr) on the temperature profile is explained in Fig. 5b. The Prandtl number describes the relationship between the thickness of the viscous and thermal boundary layers. It is evident that when the Prandtl number increases, the temperature profile also rises, even while time remains constant. Table 1 displays the change in skin friction at r = 0 and r=b for various viscosity reactive fluid values. As the viscosity of the reactive fluid rises, it is shown that the Nusselt number and skin friction both rise exponentially with the rate of heat transfer and shear stress for the plate, but not with the exponential growth of the table. Tables 3 and 4 show that both plates significantly improve when the mixed convection parameter is increased for both exponentially increasing and exponentially decaying systems. Tables 5 and 6 show that when the Prandtl number increases, the shear stress gradually decreases.

Table 1.0: Skin friction and Nusselt number when δ = -2

λ	Skin0	Skin1	Nu0	Nu1
0.1	0.0050	0.8767	0.0006	0.0031
0.2	0.0054	0.8814	0.0013	0.0061
0.3	0.0058	0.8861	0.0019	0.0092
0.4	0.0062	0.8908	0.0026	0.0123
0.5	0.0066	0.8955	0.0032	0.0154

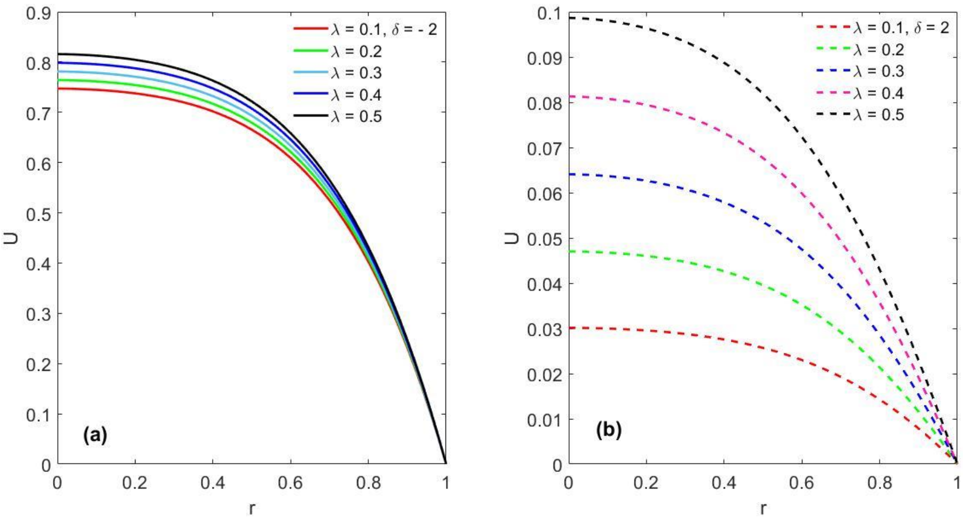

Fig 2: Velocity profile for different values of ʎ

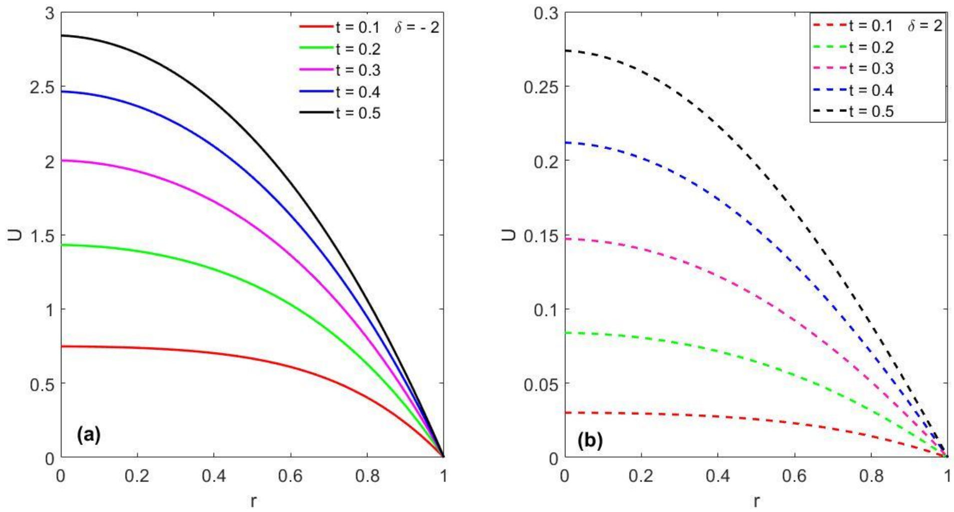

Fig 3: Velocity profile for different values of t

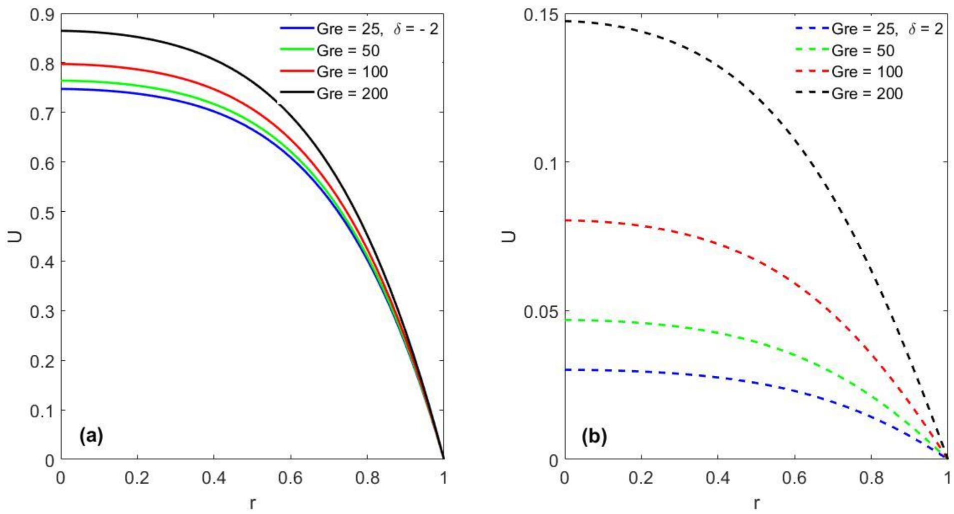

Fig 4: Velocity profile for different values of Gre

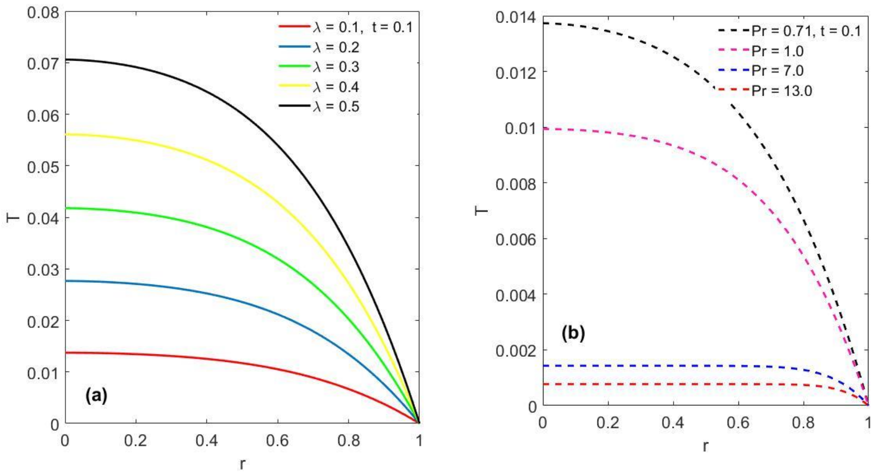

Fig 5: Temperature profile for different values of ʎ and Pr

Default values: Gre = 25, t = 0.1, Pr = 0.71, ε = 0.01, ʎ = 0.1

Table 2.0: Skin friction and Nusselt number when δ = 2

ʎ	Skin0	Skin1	Nu0	Nu1
0.1	0.0047	0.2063	0.0063	0.0306
0.2	0.0086	0.2531	0.0063	0.0306
0.3	0.0126	0.3000	0.0191	0.0920
0.4	0.0166	0.3471	0.0257	0.1228
0.5	0.0207	0.3944	0.0323	0.1536

Table 3.0: Skin friction for Gre when δ = -2 and δ =2

δ = -2			δ = 2		
Gre	**Skin0**	**Skin1**	**Gre**	**Skin0**	**Skin1**
25	0.0050	0.8767	25	0.0047	0.2063
50	0.0054	0.8814	50	0.0086	0.2529
100	0.0061	0.8807	100	0.0164	0.3461
150	0.0069	0.9000	150	0.0242	0.4393
200	0.0077	0.9094	200	0.0319	0.5325

Table 4.0: Skin friction and Nusselt number when δ = -2

Pr	Skin0	Skin1	Nu0	Nu1
0.71	0.0050	0.8767	0.0006	0.0031
1.0	0.0049	0.8764	0.0006	0.0031
7.0	0.0046	0.8738	0.0000	0.0027
13.0	0.0046	0.8731	0.0000	0.0023

Table 5.0: Skin friction and Nusselt number when δ = 2

Pr	Skin0	Skin1	Nu0	Nu1
0.71	0.0047	0.2063	0.0063	0.0306
1.0	0.0039	0.2030	0.0056	0.0306
7.0	0.0009	0.1766	0.0000	0.0265
13.0	0.0009	0.1704	0.0000	0.0231

4. CONCLUSION

The governing equation for hydrodynamic flow in vertical tube with r = 0 and r = b. has been described mathematically and numerically analyzed. Investigations have been conducted on the interactions between velocity, temperature, and the radially applied exponentially decaying/growing time-dependent pressure gradient. The problem at hand has been solved using an implicit finite difference scheme technique. Both graphic and tabular representations of the impact of changing the parameters influencing the flow have been provided. The following are the notable outcomes:

1. It is discovered that as time and the slip wall coefficient are increased, the velocity distribution significantly grows.

2. Skin friction may be increased by creating a pressure gradient that increases exponentially with time and is influenced by the wall slip coefficient.
3. By using an exponentially decaying time-dependent pressure gradient and reducing the impact of time, Dean vortex instability may be reduced.

NOMENCLATURE

Symbols	**Descriptions**
C_p	Specific heat at constant pressure
E	Activation energy
g	Acceleration due to gravity
Gr	Grashof number
Nu	Nusselt number
P	Dimensional pressure
Q	Heat reaction parameter
R	Universal gas constant
Re	Reynolds number
t'	Dimensional time
t	Dimensionless time
T'	Dimensional temperature of the fluid
T_0	Initial temperature of the fluid
U_0	Reference velocity
u'	Dimensional velocity of the fluid
u	Dimensionless velocity of the fluid
r'	Dimensional radialco-ordinate
r	Dimensionless radial co-ordinate
$\Pr$	Parandtl number
β_0	Radial magnetic field
a^*	Radius of theinner cylinder
b^*	Radius of the outer cylinder
Gre	Mixed convection parameter
μ	Viscosity / Ns/m^2
δ	Dimensionless pressure gradient
α	Thermal diffusivity
θ	Dimensionless temperature
λ	Frank-kamenetskii parameter
β	Coefficient of volume expansion for law transmission

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