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**Review Article**

# Effects of Porosity on Free Convection between Vertical Walls with Point/Line Heat Source/Sink

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## Abstract

This study investigates the conjugate effects of Darcy parameter and point/line heat source/sink on natural convective, electrically conductive laminar fluid between two infinite vertical plates. The closed form solution to the resulting governing equations is been solved analytically using Laplace transform method, the effect of various parameters embedded in the flow is discussed with aid of line graphs and Tables. During the course of computation, excellent result was found.

**Keywords:** Laplace transform, laminar flow, porosity, heat source.



## 1. INTRODUCTION

The natural convection flow under the effect of applied magnetic field of an electrically conducting, laminar fluid flow has achieved a considerable interest due to the application in various fields of engineering such as, design of Magnetohydrodynamic power generators, nuclear fuel debris treatment, Ship flow meters, Magnetohydrodynamic pumps, electromagnetic material, it has a wide range of applications in the field of astrophysics, geophysics, aerodynamics, solar energy collector and plasma MHD jet flow [1].

A porous medium (or a porous material) is a material containing pores (voids). The skeletal portion of the material is often called the "matrix" or "frame." The pores are typically filled with a fluid (liquid or gas). The skeletal material is usually a solid, but structures like foams are often also usefully analyzed using concept of porous media.

The flow problem through a porous media finds many applications in geophysical and geothermal applications. These applications include spreading of chemical pollutants in water-saturated soil, disposal of nuclear wastes, migration of moisture in fibrous insulation, clean-up of refineries and extraction of geothermal energy, gas assisted injection molding, die filling processes [2] made an analysis on forced convection in a channel filled with a Brinkman-Darcy porous medium [3]. Have studied an exact solution for forced convection in a channel filled with a porous medium. The flow characteristics of Oberbeck convection of a couple stress fluid in a vertical porous stratum was studied by [4, 5]. Studied on the flow of two-immiscible fluids in porous and non-porous channel [6]. Investigated the unsteady oscillatory flow and heat transfer in a horizontal composite porous medium in the presence of viscous dissipation [7], addressed the transient, non-Darcian effects of laminar natural convection flow in a vertical channel partially filled with porous medium [8]. Studied the fully developed MHD mixed convection flow in a vertical channel partially filled with fluid and partially filled with a fluid-saturated porous medium.

The effect of heat source on mixed convection flow of a viscous incompressible and electrically conducting fluid along an infinite vertical plate with magnetic field has been studied by [9, 10]. Investigated the influence of variable thermal conductivity and heat.

Source/sink on electrically conducting and viscous incompressible fluid flow near stagnation point on a stretching sheet with magnetic field. The influence of an applied magnetic field and thermal radiation on unsteady hydromagnetic boundary layer flow with heat and mass transfer over a shrinking sheet in the presence of heat source/sink has been investigated by [11, 4] studied the influence of heat source/sink on steady combined convection flow and heat transfer of a viscous incompressible and electrically conducting fluid between vertical parallel walls with viscous dissipation [12], has used the Cattaneo-Christov theory to investigate the effect of viscous dissipation on non-Newtonian hydromagnetic fluid with non-uniform heat source/sink [1]. Analyzed the influence of constant point/line heat source on fully developed free convective laminar flow between two infinite vertical walls of an electrically conducting and viscous incompressible fluid.

The present work is motivated to study the conjugate effects of Darcy parameter and point/line heat source/sink.

## 2. MATHEMATICAL ANALYSIS

Consider a steady natural convective, fully developed flow of a viscous electrically conductive incompressible fluid in an infinite vertical channel with point/line heat source/sink. The flow geometry is considered in such away that $x-$ axis is in vertical direction while the $y-$ axis is orthogonal to both walls with origin lies on wall at $y=0$. The temperature of both walls are taken same as $T_c$. An applied magnetic field of strength $B_o$ is taken in $x-$ direction. For the formulation of the point heat source/sink [1], defined a function $F(y',a',b')$ Using the Heaviside step function as follows:

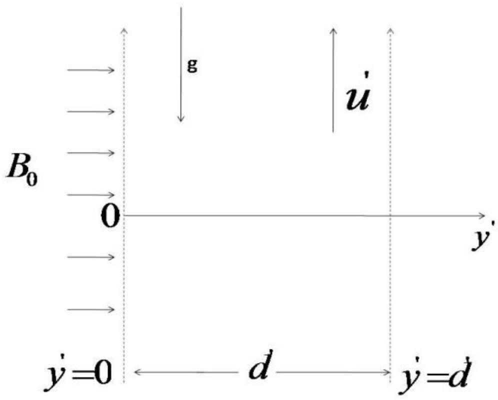


**Fig 1: Geometry of the problem**

$$H(y',a',b')=H(y'-a')-H(y'-b')=\begin{cases}0, y'<a'\\1, a'\le y'<b'\\0, y'\le b'\end{cases} \quad \text{------------------ (1)}$$

Under these assumptions and usual Boussinesq's approximation, the governing equations are written as follows:

$$\frac{\nu d^2u'}{dy'^2}-\frac{\sigma\beta_o^2u'}{\rho}-\frac{\nu u'}{\kappa'}+g\beta(T'-T_c)=0 \quad \text{-------------------------------------- (2)}$$

$$\frac{\kappa d^2T'}{dy'^2}+Q_oF(y',a',b')=0 \quad \text{-------------------------------------------------------- (3)}$$

The governing conditions according to the considered model are given as follows:

$$u'=0, T'=T_c \text{ at } y'=0 \quad \text{------------------------------------------------------------- (4)}$$

$$u'=0, T'=T_c \text{ at } y^*=0 \quad \text{------------------------------------------------------------ (5)}$$

For obtaining the dimensionless forms of Equations (2-5), we use the non-dimensional variables which are as follows:

$$u=\frac{u'}{u_o}, y=\frac{y'}{d'}, u_o=\frac{g\beta d'^2 T_c}{T_c}, a=\frac{a'}{d'}, b=\frac{b'}{d'}, Da=\frac{K'}{d'^2}, \theta=\frac{T'-T_c}{T_c} \quad \text{------------- (6)}$$

Applying Equation (6) to Equations (1-5), we have

$$\frac{d^2u}{dy^2}-\left(M^2+\frac{1}{Da}\right)u+Gr\theta=0 \quad \text{------------- (7)}$$

$$\frac{d^2\theta}{dy^2}+\alpha F(y,a,b)=0 \quad \text{------------- (8)}$$

$$H(y,a,b)=H(y-a)-H(y-b)=\begin{Bmatrix} 0, y<a \\ 1, a\le y<b \\ 0, y\le b \end{Bmatrix}, 0<a<b<1. \quad \text{------------- (9)}$$

$$u=0, \theta=0 \text{ at } y=0 \quad \text{------------- (10)}$$

$$u=0, \theta=0 \text{ at } y=1 \quad \text{------------- (11)}$$

Applying Laplace transform to both sides of Equation (7-8) and solving the resulting Ordinary differential Equations with the corresponding boundary conditions, we obtain

$$\bar{\theta}=\frac{\theta_y(0)}{S^2}-\alpha\left(\frac{e^{-aS}-e^{-bS}}{S^3}\right) \quad \text{------------- (12)}$$

$$\bar{u}=\frac{u_y(0)}{S^2-a_1^2}-\frac{Gr\bar{\theta}}{S^2-a_1^2} \quad \text{------------- (13)}$$

Taking Laplace Inverse transform to both sides of Equations (12-13) and using partial fraction technique, we have

$$u(y)=\frac{u_y(0)}{a_1}Sinha_1 y-\frac{\theta_y(0)}{a_1^3}\left(sinha_1 y-a_1 y\right)+\frac{\alpha}{2a_1^4}\Big(H(y-b)\left[(y-b)^2a_1^2-2\cosh a_1(y-b)+2\right]-$$
$$H(y-a)\left[(y-a)^2a_1^2-2\cosh a_1(y-a)+2\right]\Big) \quad \text{------------- (14)}$$

$$\theta(y)=\theta_y(0)y-\frac{\alpha}{2}\left[\left(y^2-2ay+a^2\right)H(y-a)-\left(y^2-2by+b^2\right)H(y-b)\right] \quad \text{--------- (15)}$$

Where $\theta_y(0)$ and $u_y(0)$ are arbitrary constants and can be obtained with the help of boundary conditions (10) and (11).

$$\theta_y(0)=\frac{\alpha}{2}\left(a^2-b^2-2a+2b\right),$$

$$u_y(0)=\frac{\theta_y(0)}{a_1^2}-\frac{1}{a_1^3\sinh a_1}\left(\cosh a_1(a-1)-\cosh a_1(b-1)\right)$$

## 3. RESULTS AND DISCUSSION

The system of governing equations of the physical situation presented in (2) to (3) subject to boundary condition (4) and (5) is solved using Laplace transform technique, The parameters that are embedded in the present study and influenced the flow are Darcy number, Thermal grashof Number (Gr=1), Magnetic parameter and heat source/sink. To have a clear insight of the physical problem, the results are discussed graphically using line graphs and Tables.

Fig 2a and 2b are prepared to show the conjugate effects of Darcy number and point/line heat source sink on velocity profile. Fig 2a, shows clearly that the velocity profile of the fluid decreases as the length of the line heat source decreases in the presence of Darcy effect. Also Figure 2b shows that, the velocity profile of the fluid decreases as the length of the line heat source decreases in the absence of Darcy effect. From Fig. 2a and 2b, it is evidently clear that the maximum velocity is found in the case when large line heat source is taken, velocity of the fluid decreases very sharply. However, velocity decreases in Fig 2b more than that of Fig 2a where the porosity is present , we can conclude that velocity variation of the fluid between vertical parallel walls depends on the variation of arbitrary constants ( a and b) and Darcy effect.

Fig 3 and Table 2 are prepared to show the effects of Darcy number (Da) on velocity profile in the case of point heat source (a= 0.4, b = 0.5), as observed, when the permeability of the medium increases there is increase in the fluid velocity since the barriers placed on the flow path reduce as Darcy number (Da) increases allowing for free flow thus increasing the velocity and vice-versa. Fig 4 and Table 1 are prepared to capture the effects of Darcy number(Da) on velocity profile in the case of line heat source(a = 0.1, b = 0.9), it is obvious to mention that as the permeability of medium increases the velocity of the fluid increases as seen in Fig 3. But observing Fig 3 and Fig 4 carefully delineates that, the velocity increase is more pronounce in the case of point heat source compare to line heat source.

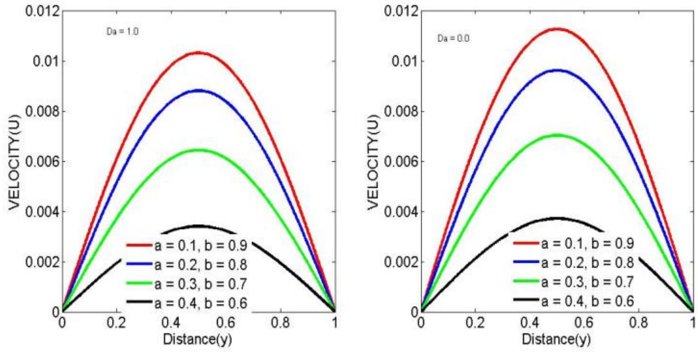


**Fig 2a and 2b: comparison between the present work and previous work of Dwivedi [1]**

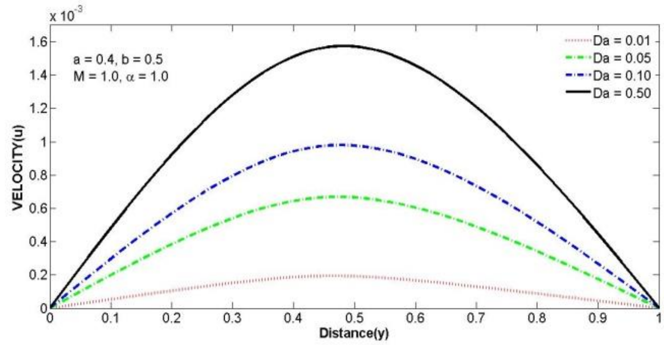


**Fig 3: Effects of Darcy number on velocity in the case of point heat source/sink**

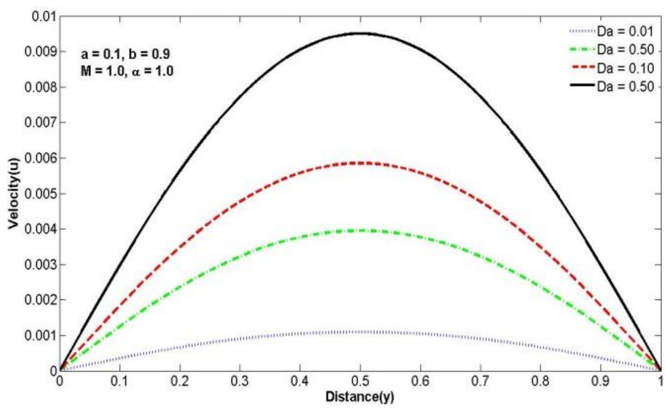


**Fig 4: Effects Darcy number (Da) on velocity in the case of Line heat source**

**Table 1: Variation of Darcy number in the case of Line heat source**

| | Dwivedi *et al., 2019* **Da** = **0**, $a$ = 0.1, $b$ = 0.9 | Present study **Da** = **0**.1, $a$ = 0.1, $b$ = 0.9 |
|---|---|---|
| $y$ | $Velocity(u)$ | $Velocity(u)$ |
| 0.0 | 0.0000 | 0.0000 |
| 0.2 | 0.0067 | 0.0035 |
| 0.4 | 0.0107 | 0.0056 |
| 0.6 | 0.0107 | 0.0056 |
| 0.8 | 0.0067 | 0.0035 |
| 1.0 | 0.0000 | 0.0000 |

**Table 2: Variation of Darcy number in the case of Point heat source**

| | Dwivedi *et al., 2019* **Da** = **0**, $a$ = 0.4, $b$ = 0.5 | Present study **Da** = **0**.1, $a$ = 0.4, $b$ = 0.5 |
|---|---|---|
| $y$ | $Velocity(u)$ | $Velocity(u)$ |
| 0.0 | 0.0000 | 0.0000 |
| 0.2 | 0.0011 | 0.0006 |
| 0.4 | 0.0018 | 0.0009 |
| 0.6 | 0.0017 | 0.0009 |
| 0.8 | 0.0010 | 0.0005 |
| 1.0 | 0.0000 | 0.0000 |

## 4. CONCLUSION

The conjugate effect of Darcy parameter and point/line heat source/sink on the steady free convective flow in a vertical walls has been examined in this work, we have obtained the following conclusions:

1. The velocity profile has increasing tendency with increasing the value of Darcy number.
2. The influence of heat source parameter is to enhance the velocity of the fluid.
3. The Darcy parameter enhances the velocity of the fluid for both cases of point heat source and line heat source.

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