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Original Research Article

MHD Slip Flow of an Exothermic Fluid in a Convectively Heated Porous Vertical Channel

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Abstract

An inspection is implemented to study MHD slip flow of viscous reactive fluid in a convectively heated porous vertical channel. The ordinary differential equation governing the flow field is solved by a semi-analytical method (perturbation series method) where suction/injection is used as a fluid controller in the channel. The results are presented graphically and discussed quantitatively with respect to various parameter embedded in the problem. It is observed that for both suction/injection, increase in Magnetic field parameter, decrease the velocity flow and skin friction.

Keywords: MHD slip flow, flow field, suction/injection, chemical reaction.

1. INTRODUCTION

Suction/injection application on flow of heat and mass transfer has built many interest of study because of its large-scale applications in engineering. The influence of suction/injection on controlling boundary layer has significantly contributed in the field of aerodynamic and space science. Shojeaefard *et al.,* [1] applied wall suction/injection to regulate fluid flow on the surface of subsonic airplane which might result to decrease in fuel consumption by 30%, a substantial cut in release of pollutant is reached, and cost of operating commercial planes are reduced by at least 8%, see the work of Braslow [2]. The role of suction/injection on steady hydro magnetic convective flow of viscous reactive fluid between vertical porous plates with thermal diffusion is analyzed by Uwanta and Hamza [3]. The suction/injection of fluid through the boundary surface in cooling of mass transfer can remarkably changes the flow fields which as a result affect the transfer rate of heat from the plate. See the study of Ishak *et al.*, [4]. Devi and Kandasamy [5] study the effect of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary-layer flow over a wedge with suction/injection.

Heat enhanced by an exothermic reaction on a fully developed MHD mixed convection flow in a vertical channel with first order chemical reaction is analyzed by Jayabalan *et al.,* [6]. The effect of exothermic fluid and transpiration on mixed convection flow in a vertical porous plate is examined by Hamza *et al.,* [7].

Asymptotic analysis of the chemical reaction and the Newtonian heating parameter is carried out by Kamran and Wiwatanapataphee [8], where the Newtonian heating parameter raises the thermal boundary layer thickness by 39.93% for the shrinking sheet. The hydrodynamic and thermal behavior of steady fully developed free and mixed confection flow in a vertical parallel plate in the present of suction/injection was analyzed by Jha *et al.,* [9] and Jha and Aina [10]. Also Jha *et al.,* [11] analyses the unsteady hydromagnetic natural convection flow near impulsive wall as accelerated motion of the infinite vertical porous plate. The influence of suction/injection at top/bottom wall on non- darcy mixed convection flow is analyzed by Murthy and Kumar [12]. Chamkha *et al.,* [13] investigate the effect of internal heat generation on free convection along a vertical plate. The concept of slip flow has received a consideratble attention because of its application in micro and Nano channels, modern sciences and technology [14]. The boundary layer flow and heat transfer of a nanofulid over a streatching sheet are numerically studied by Sharma *et al.,* [15]. The combined effect of Navier slip and Newtonian heating on MHD unsteady flow and heat transfer toward a flat plate where shown by Makinde [16]. Also free convection slip flow of an exothermic fluid in a convectively heated vertical channel was analyzed by Hamza [17]. The aim of the present analysis is to study the effect of suction/injection on MHD slip flow of viscous reactive fluid in a convectively heated porous vertical channel. In this paper suction/injection is used to control fluid flow in the channel.

2. MATHEMATICAL ANALYSIS

Consider a steady state natural convection flow of viscous reactive fluid of an Arrhenius kinetic in a channel formed by two infinite vertical parallel porous plates separated by a distance H under the influence of a transverse magnetic field of strength B_0, as shown in fig.1. It is assumed that the flow is subjected to suction of the fluid from one porous plate and at the same rate fluid is being injected through the other porous plate.

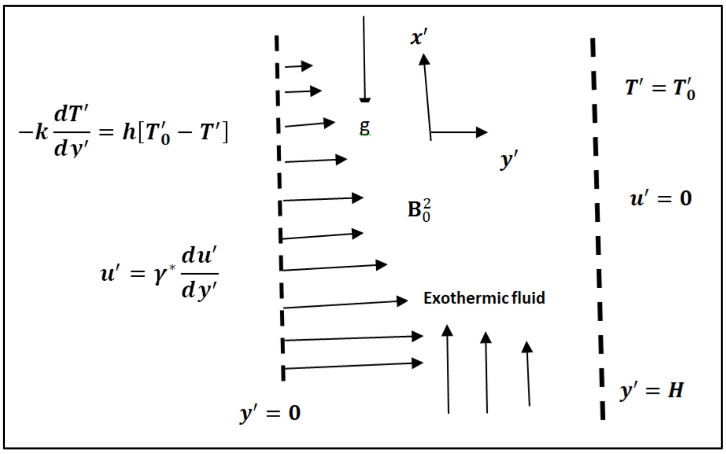

Figure 1: Geometry of the problem

The dimensional governing equations together with boundary conditions are given as;

$$v\frac{d^2u'}{dy'^2} + v_0\frac{du'}{dy'} + g\beta(T' - T'_0) - \frac{\nabla B_0^2 u'}{\rho} = 0 \quad (1)$$

$$\frac{k}{\rho C_p}\frac{d^2T'}{dy'^2} + v_0\frac{dT'}{dy'} + \frac{QC_0^*A}{\rho C_p} e^{\left(\frac{-E}{RT'}\right)} = 0 \quad (2)$$

The boundary conditions for the present problem are:

$$\left.\begin{array}{c} u' = \gamma^*\frac{du'}{dy'}, -k\frac{dT'}{dy'} = h[T'_0 - T'], at\ y' = 0 \\ u' = 0, T' = T'_0\ at\ y' = H \end{array}\right\} \quad (3$$

where β is the coefficient of thermal expansion, *Q* is the heat of reaction, *A* is the rate constant, *E* is the activation energy, *R* is the universal constant, *v* is the kinematic viscosity, C_0 is the initial concentration of the reactant species, *g* is the gravitational force, C_p is the specific heat at constant pressure and *k* is the thermal conductivity of the fluid, while ρ is the density of the fluid.

To solve equations (1) and (2), we employ the following dimensionless variables and parameters:

$$\left.\begin{array}{l} y = \frac{y'}{H}, \theta = \frac{E(T' - T_0)}{RT_0^2}, \varepsilon = \frac{RT_0}{E}, U = \frac{u'\mu_0 E}{g\beta H^2 RT_0^2}, \lambda = \frac{QC_0^*AEH^2}{RT_0^2} e^{\left(\frac{-E}{RT_0}\right)}, Pr = \frac{\mu_0\rho C_p}{k} \\ \gamma = \frac{\gamma^*}{H}, \theta_a = \frac{E(T_a - T_0)}{RT_0^2}, Br = \frac{hH}{k}, \\ S = \frac{v_0 H}{v} \end{array}\right\} \quad (4)$$

Using (4), Equations (1)-(3) take the following form:

$$\frac{d^2U}{dy^2} + S\frac{dU}{dy} - Ha^2U = -\theta \quad (5)$$

$$\frac{d^2\theta}{dy^2} + SPr\frac{d\theta}{dy} + \lambda e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0 \quad (6)$$

The corresponding boundary conditions are;

$$\left.\begin{array}{l} U - \gamma\frac{dU}{dy} = 0, \frac{d\theta}{dy} - Br[\theta - \theta_a] = 0\ at\ y = 0 \\ U = 0, \theta = 0, at\ y = 1 \end{array}\right\} \quad (7)$$

Where Pr, γ, Br, λ, θ_a , Ha, S and ε are Prandtl number, Navier slip parameter, Biot number, Frank-Kamenetskii parameter, ambient temperature, Hartmann number, Suction/Injection parameter and activation energy parameter.

3. APPROXIMATED SOLUTION

The analytical solutions have played an important role in validating and exploring computer routines of complicated problems. They are also used to inspect the internal consistency of mathematical models and of the approximations adopted by Hamza [17].

The state governing equations together with boundary condition can be written as follows:

$$\frac{d^2U}{dy^2} + S\frac{dU}{dy} - Ha^2U = -\theta \qquad (8)$$

$$\frac{d^2\theta}{dy^2} + SPr\frac{d\theta}{dy} + \lambda e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0 \qquad (9)$$

The corresponding boundary conditions are;

$$\left.\begin{array}{r} U - \gamma\frac{dU}{dy} = 0, \frac{d\theta}{dy} - Br[\theta - \theta_a] = 0 \; at \; y = 0 \\ U = 0, \theta = 0, at \; y = 1 \end{array}\right\} \qquad (10)$$

To obtain the approximate solutions of equations (8) and (9) subject to (10), we take the power series expansion in the Frank-Kamenetskii parameter λ of the form:

$$\left.\begin{array}{r} U = U_0 + \lambda U_1 + \lambda^2 U_2 + o(\lambda) \\ \theta = \theta_0 + \lambda\theta_1 + \lambda^2\theta_2 + o\,(\lambda) \end{array}\right\} \qquad (11)$$

Substituting (11) into (8)-(10) and equating the coefficient of like powers of λ, the resulting solutions of the momentum and energy balance equations are as follows:

$$U = B_{21}e^{m1y} + B_{22}e^{-m2y} + d_1 + d_2e^{-ky} + \lambda[B_{23}e^{m1y} + B_{24}e^{-m2y} + f_0 + f_1e^{-ky} + f_2y + f_3ye^{-ky} + f_4e^{-2ky} + f_5e^{-3ky}] + \lambda^2[B_{25}e^{m1y} + B_{26}e^{-m2y} + g_0 + g_1e^{-ky} + g_2y + g_3y^2 + g_4ye^{-ky} + g_5y^2e^{-ky} + g_6e^{-2ky} + g_7ye^{-2k}g_8e^{-3ky}] \qquad (12)$$

$$\theta = A_{21} + A_{22}e^{-ky} + \lambda[A_{23} + A_{24}e^{-ky} + s_0y + s_1ye^{-ky} + s_2e^{-2ky} + s_3e^{-3ky}] + \lambda^2[A_{25} + A_{26}e^{-ky} + t_0y + t_1y^2 + t_2ye^{-ky} + t_3y^2e^{-ky} + t_4e^{-2ky} + t_5ye^{-2ky} + t_6e^{-3ky}] \qquad (13)$$

From (12), the steady state skin frictions on the boundaries are as follows:

$$\frac{dU}{dy}|_{y=0} = [m1B_{21} - m2B_{22} - kd_2] + \lambda[m1B_{23} - m2B_{24} - kf_1 + f_2 + f_3 - 2kf_4 - 3kf_5] + \lambda^2[m1B_{25} - m2B_{26} - kg_1 + g_2 + g_4 - 2kg_6 + g_7 - 3kg_8] \qquad (14)$$

$$\frac{dU}{dy}|_{y=1} = [m1B_{21}e^{m1} - m2B_{22}e^{-m2} - kd_2e^{-k}] + \lambda[m1B_{23}e^{-m1} - m2B_{24}e^{-m2} - kf_1e^{-k} + f_2 + f_3[e^{-k} - ke^{-k}] - 2kf_4e^{-2k} - 3kf_5e^{-3k}] + \lambda^2[m1B_{25}e^{m1} - m2B_{26}e^{-m2} - kg_1e^{-k} + g_2 + 2g_3 + g_4[e^{-k} - ke^{-k}] + g_5[2e^{-k} - ke^{-k}] - 2kg_6e^{-2k} + g_7[e^{-2k} - 2ke^{-2k}] - 3kg_8e^{-3k}] \qquad (15)$$

Also, from (13) the rates of heat transfer on the boundaries in terms of Nusselt number are as follows:

$$\frac{d\theta}{dy}|_{y=0} = -KA_{22} + \lambda[-KA_{24} + s_0 + s_1 - 2ks_2 - 3ks_3] + \lambda^2[-kA_{26} + t_0 + t_2 - 2kt_4 + t_5 - 3kt_6] \qquad (16)$$

$$\frac{d\theta}{dy}|_{y=1} = -kA_{22}e^{-k} + \lambda[-kA_{24}e^{-k} + s_0 + s_1[e^{-k} - ke^{-k}] - 2ks_2e^{-2k} - 3ks_3e^{-3k}] + \lambda^2[-kA_{26}e^{-k} + t_0 + 2t_1 + t_2[e^{-k} - ke^{-k}] + t_3[2e^{-k} - ke^{-k}] - 2kt_4e^{-2k} + t_5[e^{-2k} - 2ke^{-2k}] - 3kt_6e^{-3k}] \qquad (17)$$

4. RESULT AND DISCUSSION

The basic parameters that govern this flow are Magnetic field parameter (*Ha*), Navier slip parameter (γ), Biot number (Br), Frank-Kamenetskii parameter (λ) and Suction/injection parameter (S). Unless stated otherwise, the following values are assign to the governing parameters, Pr=0.71, ε= 0.01, θ_a=1.

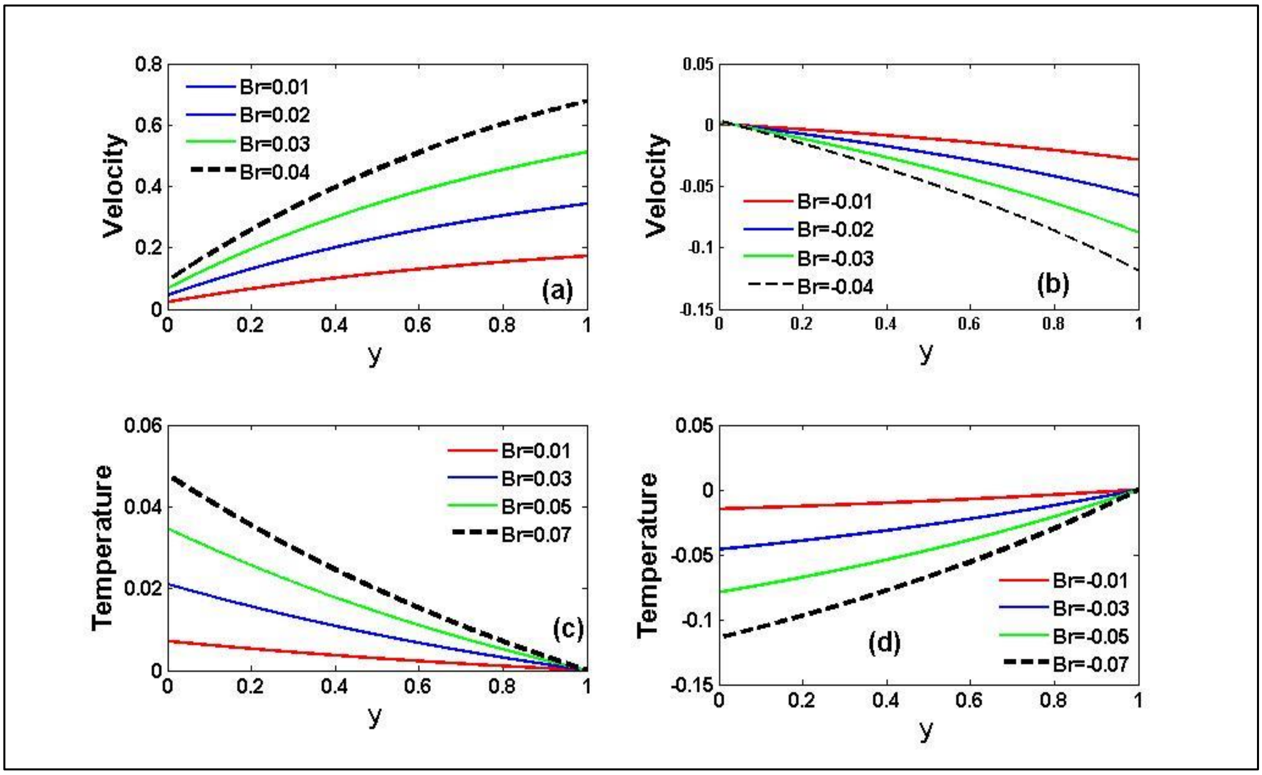

Fig. 2: Effect of Biot number (Br) on Velocity and Temperature for Injection(S=+1) and Suction(S=-1). Where Ga=0.1, Ha=0.1, \lambda=0.01

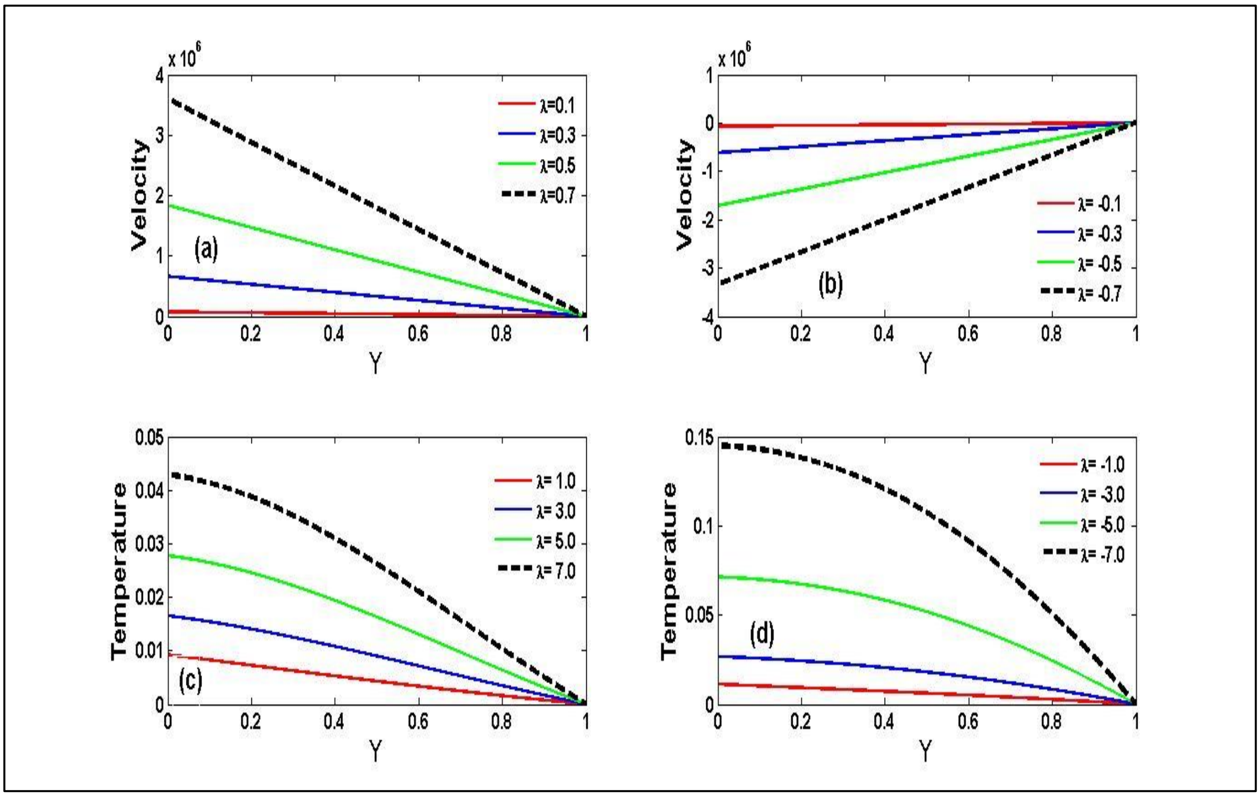

Fig. 3: Effect of Frank-Kamenetski parameter (λ) on velocity and Temperature for S=+0.01/ S=-0.01 and also S=+1/S=-1 for velocity and temperature respectively. Where γ=0.1, Ha=0.1, Br= 0.01

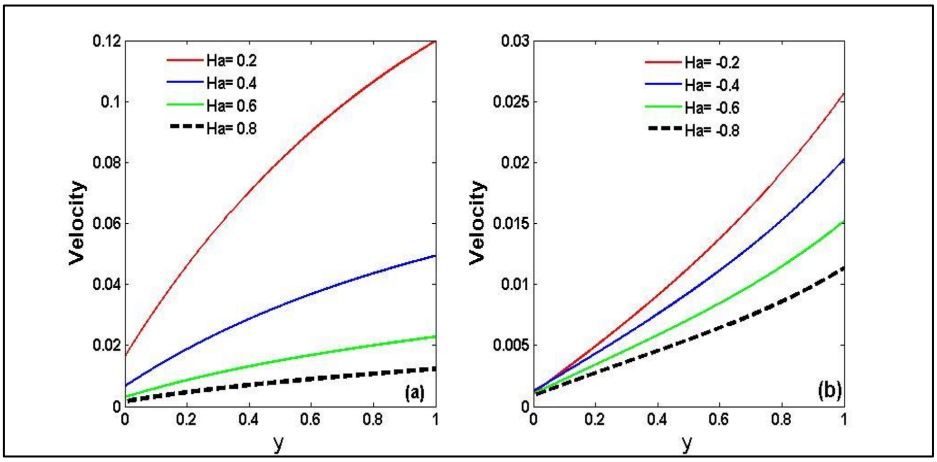

Fig. 4: Effect of Hartmann number (Ha) on velocity for injection(S=+1) and suction(S=-1)

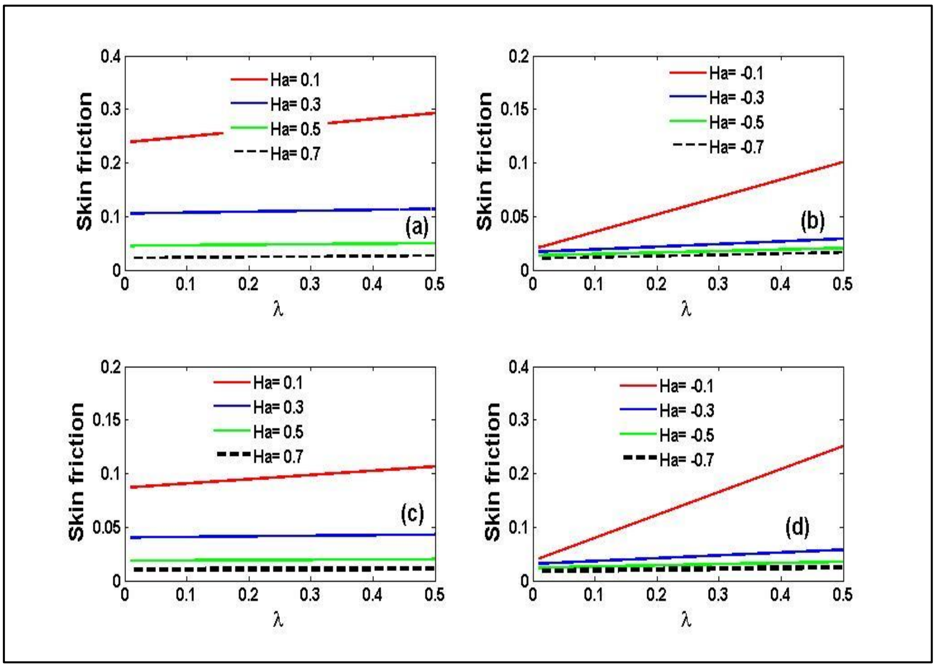

Fig. 5: Effect of Hartmann number (*Ha*) on Skin friction for distance y=0 and y=1 on injection(S=+1) and suction(S=-1) with, γ=0.1, Br=0.01.

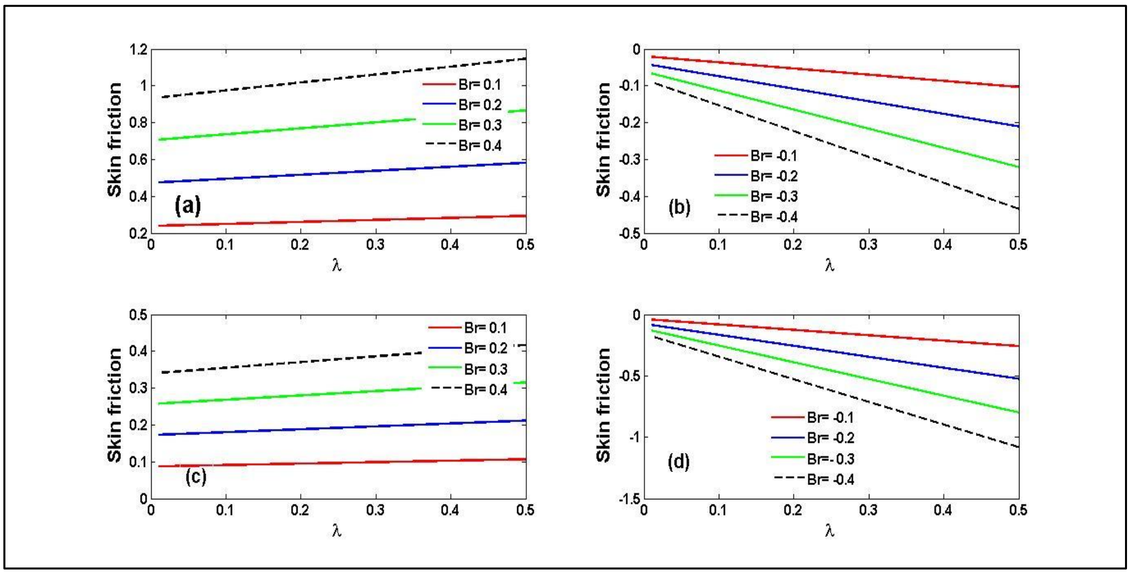

Fig. 6: Effect of Biot number (Br) on Skin friction for distance y=0 and y=1 on injection(S=+1) and suction(S=-1) with „γ=0.1, Ha=0.1

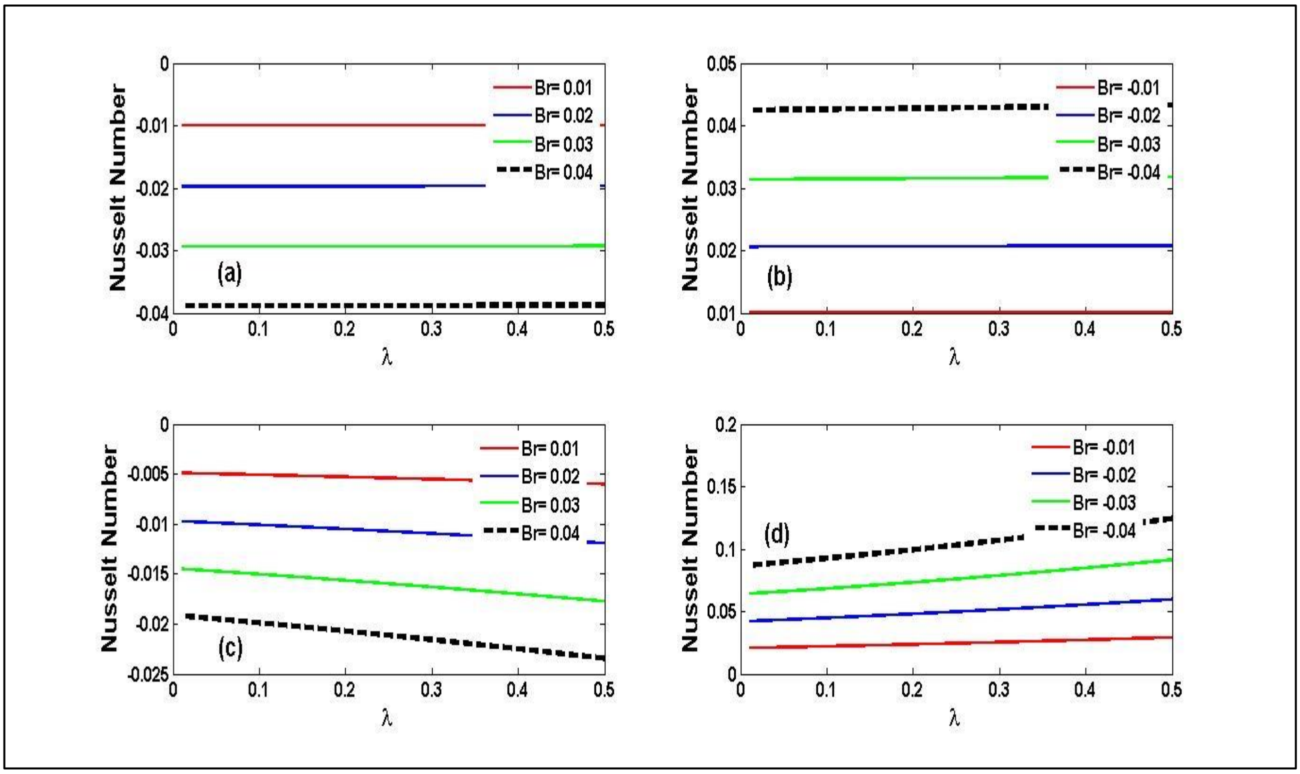

Fig. 7: Effect of Biot number (Br) on Nusselt number for distance y=0 and y=1 on injection(S=1) and suction(S=-1).

Fig. 2 show that an increase in Biot number (Br) lead to increase in velocity and temperature profile for S=+1, see Fig.2a and Fig.2c respectively. While increase in Biot number, decreases both velocity and temperature profile for S=-1. See Fig.2b and Fig.2d respectively. The influence of Frank-Kamenetskii parameter (λ) is shown in Fig.3, where fig.3a show that increase in λ increases velocity profile for S=+1 but fig.3b shows that velocity decreases when S=-1. While on the other hand, temperature increases with an increase in λ for both S=+1 and S=-1. See fig.3c and fig.3d. Also, Fig.4a and fig.4b show that an increase in Hartmann number (Ha) lead to decrease in velocity for both S=+1 and S=-1 respectively. Hartmann number (Ha) has an effect on skin friction, fig. 5 shows that increase in Ha lead to decrease in skin friction for both S=+1 and S=-1 on distance y=0 and y=1, see fig.5a & fig.5b and Fig.5c & fig.5d respectively. Influence of Biot number on skin friction for both distance is shown in fig.6, fig.6a and fig.6c respectively shows that increase in Biot number (Br) for S=+1 on both distance y=0 and y=1 lead to increase in skin friction profile. While for S=-1, the skin friction decreases with increase in Br on both distance y=0 and y=1, see fig.6b and fig.6d respectively. Finally, Fig.7a and c respectively shows the rate of heat transfer decreases with an increase in Biot number (Br) for S=+1 on both distance y=0 and y=1. While fig.7b and d shows an increase in rate of heat transfer with an increase in Br for S=-1 on both distance y=0 and y=1 respectively.

5. CONCLUSION

The present paper investigates the effect of MHD slip flow of exothermic fluid in a convectively heated porous vertical channel. The approximated solution of the problem is obtained using perturbation method. Summary of the major findings are as follows:

i. An increase in Biot number lead to an increase in temperature, velocity and skin friction for S=+ve but decreases for S=-ve.
ii. Frank Kamenetskii parameter rises, increase the temperature profile for both S=+ve and S=-ve.
iii. Skin friction and Velocity decreases with an increase in Hartmann number for both S=+ve and S=-ve.
iv. The rate of heat transfer decrease for every increase in Br at distance y=0 for S=+ve but the heat transfer increase when distance y=1 for S=-ve.

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Appendix A

$$k = SPr, A_{22} = \frac{-Br\theta_a}{Br(e^{-k}-1)-k}, A_{21} = -A_{22}e^{-k}, r_0 = -A_{21} + \varepsilon A_{21}^2 - \frac{\varepsilon^2 A_{21}^3}{2}$$

$$r1 = -A_{22} + 2\varepsilon A_{21}A_{22} - \frac{3S^2 {A_{21}}^2 A_{22}}{2}, r2 = -\varepsilon A_{22}^2 - \frac{3S^2 A_{21} A_{22}^2}{2}, r3 = \frac{S^2 A_{22}^3}{2}$$

$$s_0 = \frac{r_0}{k}, s_1 = \frac{-r_1}{k}, s_2 = \frac{r_2}{2k^2}, s_3 = \frac{r_3}{6k^2}, j_0 = 1 + Br, j_1 = 1 + Bre^{-k}$$

$$j_1 = 1 + Bre^{-k}, j_2 = 2k + Br(1-e^{-2k}), j_3 = 3k + Br(1-e^{-3k})$$

$$A_{24} = \frac{-s_0 j_0 - s_1 j_1 + s_2 j_2 + s_3 j_3}{Br(e^{-k}-1)-k}, A_{23} = -[A_{24}e^{-k} + s_0 + s_1 e^{-k} + s_2 e^{-2k} + s_3 e^{-3k}]$$

$$n_0 = -A_{23} + 2\varepsilon(A_{21}A_{23}) + \frac{3\varepsilon^2 A_{21}^2 A_{23}}{2}, n_1 = -s_0 + 2\varepsilon(A_{21}s_0) + \frac{3\varepsilon^2 A_{21}^2 s_0}{2}$$

$$n_2 = -A_{24} + 2\varepsilon(A_{21}A_{24} + A_{22}A_{23}) + \frac{3S^2(A_{21}^2 A_{24} + 2A_{21}A_{22}A_{23})}{2}$$

$$n_3 = -s_1 + 2\varepsilon(A_{21}s_1 + A_{22}s_0) + \frac{3\varepsilon^2(A_{21}^2 s_1 + 2A_{21}A_{22}s_0)}{2},$$

$$n_4 = -s_2 + 2\varepsilon(A_{21}s_2 + A_{22}A_4) + \frac{3\varepsilon^2(A_{21}^2 s_2 + 2A_{21}A_{22}A_{24} + A_{22}^2 A_{23})}{2}$$

$$n_5 = 2\varepsilon(A_{22}s_1) + \frac{3\varepsilon^2(A_{21}A_{22}S_1 + A_{22}^2 s_0)}{2}$$

$$n_6 = -s_3 + 2\varepsilon(A_{21}s_3 + A_{22}s_2) + \frac{3\varepsilon^2(A_{21}^2 s_3 + 2A_{21}A_{22}s_2 + A_{22}^2 A_{24})}{2}$$

$$t_6=\frac{n_6}{6k^2},t_5=\frac{n_5}{2k^2},t_4=\frac{n_4-3kt_5}{2k^2},t_3=\frac{-n_3}{2k},t_2=\frac{2t_5-n_2}{k},t_1=\frac{n_1}{2k},t_0=\frac{n_0-2t_1}{k}$$

$$A_{26}=\frac{-t_0(1+Br)-t_1Br-t_2(1+Bre^{-k})-t_3Bre^{-k}-t_4\left(e^{-2k}-(2k+Br)\right)-t_5(Bre^{-2k}+1)-t_6(Bre^{-3k}-(3k+Br))}{-k-Br+Bre^{-k}}$$

$$A_{25}=-[A_{26}e^{-k}+t_0+t_1+(t_2+t_3)e^{-k}+(t_4+t_5)e^{-2k}+t_6e^{-3k}]$$

$$m1=\frac{-S+\sqrt{S^2+4Ha^2}}{2},m2=\frac{-(S+\sqrt{S^2+4Ha^2})}{2},d1=\frac{A_{21}}{Ha^2},d2=\frac{-A_{22}}{k^2-Sk-Ha^2}$$

$$B_{21}=\frac{d1(\gamma m2)+d2(e^{-k}(1+\gamma m2)-(1+\gamma k))}{-e^{m1}(1+\gamma m2)+(1-\gamma m1)}$$

$$B_{22}=-[d1+d2e^{-k}+B_{21}e^{m1}]$$

$$f_5=\frac{-s_3}{9k^2-3kS-Ha^2},f_4=\frac{-s_2}{4k^2-2kS-Ha^2},f_3=\frac{-s_1}{k^2-kS-Ha^2},f_2=\frac{s_0}{Ha^2}$$

$$f_1=\frac{-A_4+f_3(2k-S)}{k^2-kS-Ha^2},f_0=\frac{A_3+Sf_2}{Ha^2},$$

$$b11=f_0[e^{m2}(1+\gamma m2)-1]+f_1[e^{-k+m2}(1+\gamma m2)-(1+\gamma k)]+f_2[e^{m2}(1+\gamma m2)+\gamma]$$

$$b12=f_3[e^{-k+m2}(1+\gamma m2)+\gamma]+f_4[e^{-2k+m2}(1+\gamma m2)-(1+2\gamma k)]+f_5[e^{-3k+m2}(1+\gamma m2)+(1+3\gamma k)]$$

$$B_{23}=\frac{b11+b12}{-e^{m2+m1}(1+\gamma m1)+(1-\gamma m1)}$$

$$B_{24}=-[B_{23}e^{m1}+f_0\quad+f_1e^{-k}+f_2\quad+f_3e^{-2k}+f_4e^{-3k}]e^{m2}$$

$$g_8=\frac{-t_6}{9k^2-3kS-Ha^2},g_7=\frac{-t_5}{4k^2-2kS-Ha^2}$$

$$g_6=\frac{-t_4-g_7(S-4k)}{2k^2-2kS-Ha^2},g_5=\frac{-t_3}{k^2-kS-Ha^2},g_4=\frac{-t_2-g_5(2S-4k)}{k^2-kS-Ha^2},g_3=\frac{t_1}{Ha^2}$$

$$g_2=\frac{2Sg_3+t_0}{Ha^2},g_1=\frac{-(A_6+g_4(S-2k)+2g_5)}{k^2-kS-Ha^2}g_0=\frac{2g_3+Sg_2+A_5}{Ha^2}$$

$$w_{20}=g_0[e^{m2}(1+\gamma m2)-1]+g_1[e^{-k+m2}(1+\gamma m2)-(1+\gamma k)]+g_2[e^{m2}(1+\gamma m2)+\gamma]$$

$$w_{21}=g_3[e^{m2}(1+\gamma m2)]+g_4[e^{-k+m2}(1+\gamma m2)+\gamma]+g_5[e^{-k+m2}(1+\gamma m2)]$$

$$w_{22}=g_6[e^{-2k+m2}(1+\gamma m2)-(1+2\gamma k)]+g_7[e^{-2k+m2}(1+\gamma m2)+\gamma]+g_8[e^{-3k+m2}(1+\gamma m2)-(1+3\gamma k)]$$

$$B_{25}=\frac{w_{20}+w_{21}+w_{22}}{(1-\gamma m1)+e^{m1+m2}(1+\gamma m2)}$$

$$B_{26}=-[B_{25}e^{m1}+g_0+g_2+g_3+(g_1+g_4+g_5)e^{-k}+(g_6+g_7)e^{-2k}+g_8e^{-3k}]e^{m2}$$