

Predicting the Brittleness Ratio of Low-Density Polyethylene (LDPE) Modified Bituminous Concrete Using Mixture Regression Model

Akpan Uwem Emmanuel¹, Ohwerhi Kelly Erhiferhi^{1*}, Eme Dennis Budu¹, Nwaobakata Chukwuemeka¹

¹Department of Civil Engineering, University of Port Harcourt, Port Harcourt, Nigeria

DOI: <https://doi.org/10.36348/sjce.2025.v09i06.005>

| Received: 04.05.2025 | Accepted: 09.06.2025 | Published: 26.06.2025

*Corresponding author: Ohwerhi Kelly Erhiferhi

Department of Civil Engineering, University of Port Harcourt, Port Harcourt, Nigeria

Abstract

This study examines the effect of Low-Density Polyethylene (LDPE) modification on the brittleness and stiffness of bituminous concrete mixtures and develops a regression model to predict the brittleness ratio (S_R). Experimental results reveal that adding LDPE significantly increases stiffness, with brittleness ratios ranging from 1.998 to 3.169. While higher stiffness improves resistance to permanent deformation and rutting, brittleness ratios above the critical limit of 3.0 indicate a risk of premature cracking, especially at low temperatures. This highlights the need to optimize LDPE content to balance stiffness and ductility. A regression model was created to predict brittleness based on proportions of granite (Z_1), sand (Z_2), bitumen (Z_3), and LDPE (Z_4). The model showed good statistical validity, with an F-calculated value of 1.382 (below the critical 2.48 at 5% significance) and an R^2 of 70.38%, explaining a significant portion of brittleness variability. The Mean Absolute Percentage Deviation of 7.2% between predicted and experimental values confirms its accuracy within acceptable engineering limits. These findings suggest that LDPE-modified bituminous concrete offers improved mechanical performance but requires careful LDPE dosage control to avoid excessive brittleness. The validated model serves as a practical tool for designing durable, sustainable asphalt pavements balancing stiffness and flexibility across diverse environments.

Keywords: Brittleness ratio, low-density polyethylene, bituminous concrete, mixture regression model, stiffness.

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1. INTRODUCTION

The brittleness ratio, defined as the ratio of compressive strength to tensile strength, serves as a critical indicator of a material's ductility and fracture behavior. A higher brittleness ratio typically reflects a material's propensity for brittle failure, characterized by sudden fracture with minimal plastic deformation, whereas a lower ratio suggests greater ductility and the ability to absorb more energy before cracking or structural failure (Hucka & Das, 1974). Widely denoted as B1 in both rock and concrete studies, this parameter has long been foundational in evaluating mechanical behavior across disciplines. In rock mechanics, for example, Altindag (2003) demonstrated that drillability and fracture resistance of rocks correlated strongly with the brittleness ratio, particularly in rotary drilling applications. More recently, Zhang et al. (2020) introduced advanced approaches using Weibull statistical models to assess brittleness, emphasizing its importance in constitutive modeling and failure prediction frameworks.

In the context of bituminous concrete, assessing the brittleness ratio is especially crucial. This material is frequently subjected to dynamic traffic loads and extreme temperature variations, both of which can induce tensile and flexural stresses that lead to premature cracking. A high brittleness ratio in bituminous mixtures often signals inadequate ductility, making the pavement more vulnerable to early-stage cracking, raveling, and structural fatigue. Consequently, predicting and optimizing this ratio is key to enhancing pavement durability and lifecycle performance.

Recent efforts in sustainable pavement design have focused on incorporating recycled polymers, particularly low-density polyethylene (LDPE), into asphalt mixtures. As a widely available plastic waste product, LDPE not only addresses environmental concerns but also offers the potential to improve mechanical performance. Its integration into bituminous concrete has been shown to enhance resistance to deformation and cracking, contributing to extended

pavement service life (Al-Hadidy & Tan, 2009). These improvements align with global initiatives to reduce plastic pollution and promote sustainable infrastructure development.

Several studies underscore the beneficial role of LDPE in mitigating brittleness in asphalt mixtures. For instance, Al-Hadidy and Tan (2009) observed that pyrolyzed LDPE enhanced the strength and stability of asphalt paving materials, indicating a favorable reduction in brittleness. Similarly, Nouali *et al.*, (2024) reported that asphalt mixtures modified with both post-consumer and post-industrial LDPE exhibited enhanced resistance to moisture damage and permanent deformation, performance traits intimately linked to lower brittleness.

Given the complex interplay of material constituents and environmental influences, researchers are increasingly leveraging advanced statistical modeling techniques to predict performance outcomes. Among these, mixture regression models have proven especially effective. These models facilitate the prediction of brittleness ratio by accounting for multiple variables simultaneously, such as LDPE content, aggregate gradation, and binder content. By modeling these interactions, mixture regression provides a data-driven framework for optimizing mix design and ensuring robust performance of LDPE-modified asphalt mixtures. As such, it supports broader adoption of recycled materials in civil engineering, combining

mechanical efficiency with environmental responsibility. Hence, this study employed mixture regression model for predicting the brittleness ratio (strength ratio) of LDPE modified bituminous concrete, with LDPE content, aggregate gradation and binder content as the predictor variables.

2. MATERIALS AND METHODS

2.1 Materials

The major materials used in this research are;

Aggregates; Crushed granite with a nominal maximum size of 12.5 mm was employed as the coarse aggregate. It was obtained from a local building materials supplier in Choba, Port Harcourt, who indicated that the material originated from the Akamkpa quarry site in Calabar. For the fine aggregate, natural river sand was used, sourced from a sand fill location along the New Calabar River in the Choba vicinity. After collection, the sand was air-dried and sieved using a 4.75 mm mesh to eliminate debris and impurities. The physical characteristics of the aggregates are presented in Table 1. As shown in the table, both the coarse and fine aggregates satisfied standard grading criteria, with their coefficients of uniformity (Cu) and curvature (Cc) falling within recommended limits. The granite and sand recorded fineness modulus values of 4.25 and 3.54, respectively, confirming the suitability of the sand for construction purposes. The specific gravity values were found to be 2.77 for granite and 2.43 for the river sand.

Table 1: Properties of Aggregates (Crushed granite and River sand)

Properties	Classification Parameter	Description	Specification/ Code
1. Gradation Granite Fine River sand	Cu = 1.82, Cc= 0.82 Cu = 3.947, Cc= 1.12	Both aggregates are classified as uniformly graded because $Cu < 6$ for fine river sand and $Cu < 4$ for granite	ASTM (2006)
2. Zone of River sand	Zone II	Suitable for construction purposes	IS: 383-(1970)
3. Fineness Modulus Granite Fine River Sand	Fineness modulus = 4.25 Fineness modulus = 3.54	- Coarse sand (Ok for construction purposes)	IS:2386-(1963)
3. Specific gravity Granite Fine River sand	Gs = 2.77 Gs = 2.43	-	ASTM C128 (2015)

Bitumen;

Bitumen, also known as asphalt cement, was employed as the primary binding agent in the formulation of the bituminous mixtures. It acted as the base material for the incorporation of HDPE. The bitumen used was of the 60/70 penetration grade and was sourced from a trusted supplier located at the Mile 3 market in Port Harcourt. As reported by the supplier, the bitumen had a specific gravity of 1.09, a softening point of 53°C, a penetration value of 68, and a flash point of 250°C.

Low-density polyethylene (LDPE);

Discarded water sachets, identified as low-density polyethylene (LDPE), were utilized as the LDPE source in this study. These waste sachets were gathered from a dumpsite in Choba, Port Harcourt. Following collection, they were thoroughly cleaned and shredded to enhance their meltability during the preparation of the bituminous mixtures.

2.2 METHODS

2.2.1 Preliminary Design and Investigation

Determination of Optimum Aggregate Blend (OAB);

The aggregate blending process employed the analytical or formula-based method, utilizing linear

programming techniques implemented through Excel Solver to proportion fine river sand and granite. The resulting particle size distributions were evaluated against the standard job mix specifications provided by FMWH (2016). The general mathematical expression for blending two aggregates is shown in Equation (1).

$$aA + bB = P \quad (1)$$

In this context, a and b represent the fractional proportions of two distinct aggregates, granite (A) and fine river sand (B), that pass through a given sieve size. P denotes the combined gradation resulting from these aggregates, which should either satisfy the specified job mix formula or closely approximate the midpoint of the target gradation range. A primary constraint in the blending process is that the sum of the proportions a and b must equal 1, as indicated in Equation (2).

$$a + b = 1 \quad (2)$$

For n number of sieve sizes, Equation (1) becomes;

$$\begin{aligned} aA_1 + bB_1 &= P_1 \\ aA_2 + bB_2 &= P_2 \\ aA_3 + bB_3 &= P_3 \\ &\vdots \\ aA_n + bB_n &= P_n \end{aligned} \quad (3)$$

Manually calculating the values of a and b from Equation (3) is notably complex and impractical. As a result, the use of efficient computational tools is essential for this process. In this study, Excel's Solver tool was employed to determine the Optimal Aggregate Blend

(OAB). The following procedures were carried out to perform the OAB optimization using Excel Solver;

- i. The mid-point of the specified job mix for each sieve size was determined
- ii. Sieve analysis was conducted on the different aggregates to obtain the gradation (percent finer than each) of the considered sieve size
- iii. An initial or arbitrary value of a and b depending on the number of aggregates considered in that category was chosen
- iv. Equation (1) was then applied to obtain the grading (P) for each of the considered sieve size
- v. The difference between P and the mid-point of the job mix was calculated for the different sieve sizes
- vi. These differences were then squared to obtain the square of differences (SD)
- vii. The sum of squares of these differences (SSD) was then obtained
- viii. Then finally excel solver was then employed for the optimization process with;
 - Objective function; Minimizing (SSD)
 - By changing variables; A and B
 - Subjecting to constraints; $a + b = 1$
 - $a, b, A, B \geq 0$

The outlined steps were meticulously followed to achieve the optimum aggregate blend for the dense asphalt concrete mixtures. Upon optimization, the proportion of aggregate A was determined to be 54.5% (a), while aggregate B accounted for 45.5% (b). The final blend conforms to the requirements of the specified job mix formula, as illustrated in **Figure 1**.

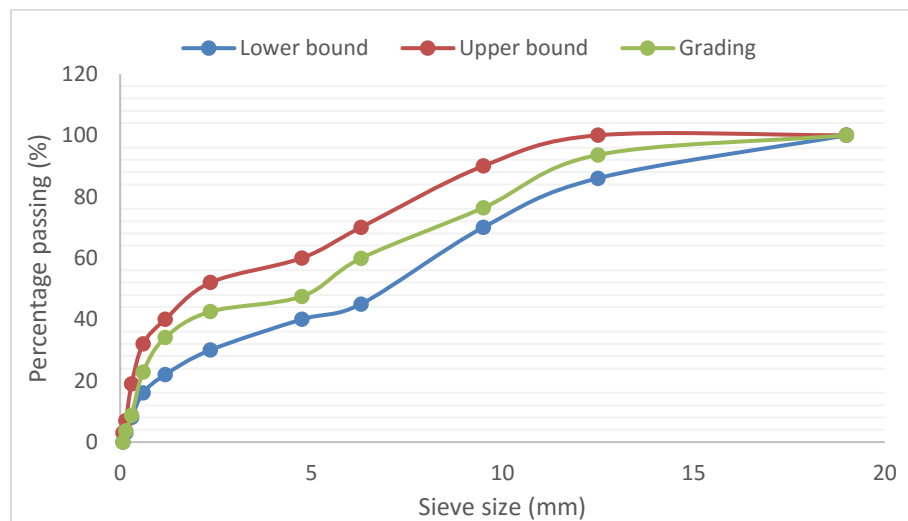


Figure 1: Gradation curves for OAB in Accordance to FMWH (2016)

Determination of Optimum Bitumen Content (OBC);

The preliminary wet mix design followed the Marshall standard procedure to establish the optimum bitumen content. Specimens were compacted with 50 blows on each side, suitable for medium traffic conditions, using a compaction hammer with a 450 mm free fall. Samples were prepared at a controlled

temperature of 60°C prior to conducting density-void analysis and stability-flow tests. The bitumen content was initially set between 4% and 6% of the total mix weight, with adjustments made according to the determined optimum aggregate blend (OAB). For each trial, the bitumen content was incrementally increased by 0.5%, and the procedure was repeated. Graphical

analyses of density-void and stability-flow data were used to determine the optimum bitumen content (OBC) for the unmodified bituminous concrete mixtures, focusing on the relationships between bitumen content and corrected Marshall stability, voids in the total mix (VTM), and unit weight, as shown in Figures 2 through 4. Applying Equation (4), the OBC was calculated to be

5.95%, which served as the basis for the main experimental mix design.

$$B_0 = \frac{B_1 + B_2 + B_3}{3} \quad (4)$$

Where; B_0 = Optimum Bitumen Content, B_1 = Percent bitumen content at maximum stability, B_2 = Percent bitumen content at maximum unit weight, B_3 = Percent bitumen content at 4% air voids in the total mix.

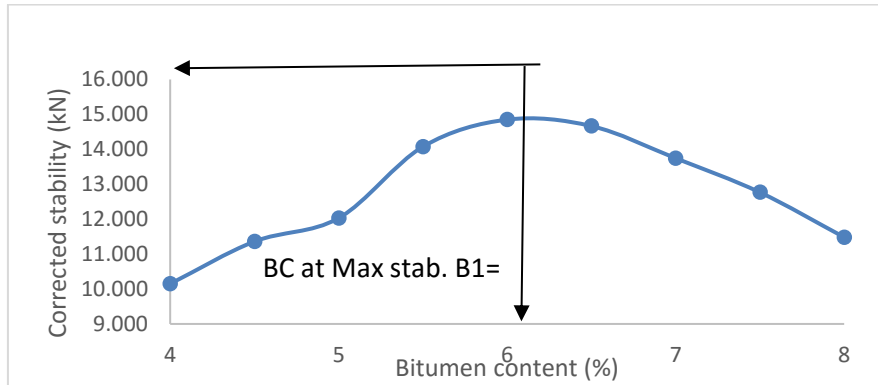


Figure 2: Stability vs % bitumen of unmodified bituminous mixtures

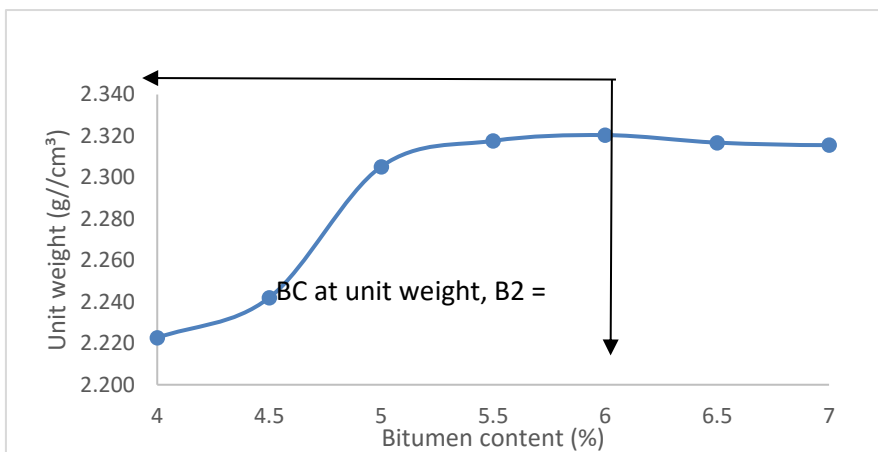


Figure 3: Unit weight vs % bitumen of unmodified bituminous mixtures

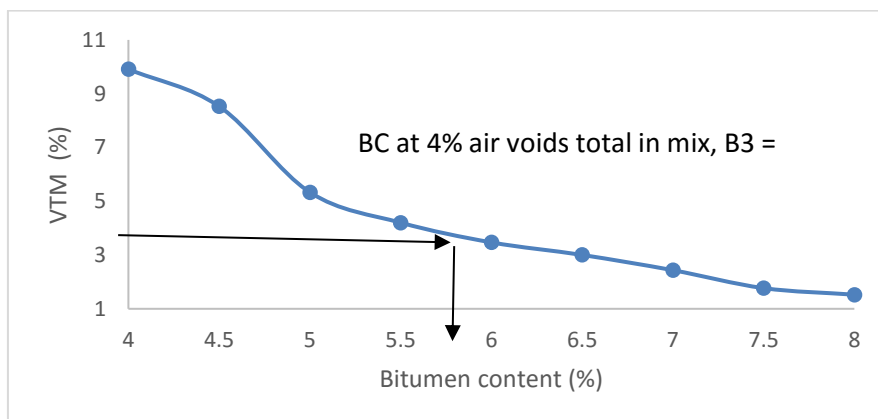


Figure 4: Percent air voids, VTM vs % bitumen of unmodified bituminous mixtures

2.2.2 Main Experimental Design

For designing the experiment involving LDPE-modified bituminous mixtures, the extreme vertices design method was applied using Minitab software.

LDPE was incorporated as a partial replacement for the bitumen content. The optimal mix proportions for the bituminous concrete without LDPE were determined to be 5.95% bitumen, 51.26% granite, and 42.79% fine

river sand by weight of the total mixture. Based on previous studies and practical experience, LDPE can replace bitumen content up to approximately 10%, which corresponds to a maximum LDPE content of about 0.6% by total mix weight. Since the introduction of LDPE could influence the proportions of other components such as granite and sand, their quantities were adjusted accordingly. Consequently, LDPE content ranged from 0

to 0.6% by weight of the total mix, bitumen varied between 5.35% and 5.95%, granite ranged from 51.26% to 51.58%, and sand varied between 42.79% and 43.07%. The specified lower and upper limits for each material are detailed in Table 2. Based on these constraints, Minitab generated 15 different combinations of LDPE-modified bituminous mixture constituents, as presented in Table 3.

Table 2: LDPE-modified Bituminous Mixtures Boundary Specifics for Mixture Design Development

Constraints	Materials/constituents			
	Granite (%)	Sand (%)	Bitumen (%)	LDPE (%)
Lower bound	51.26	42.79	5.35	0
Upper bound	51.58	43.07	5.95	0.6

Table 3: Mixture Design for LDPE-modified Bituminous Mixtures Production

Run order	Granite (%)	Sand (%)	Bitumen (%)	LDPE (%)
1	51.48857	42.85	5.435714	0.225714
2	51.48857	42.99	5.435714	0.085714
3	51.32857	42.99	5.435714	0.245714
4	51.48857	42.85	5.575714	0.085714
5	51.26	42.79	5.35	0.6
6	51.26	43.07	5.67	0
7	51.32857	42.85	5.735714	0.085714
8	51.26	42.79	5.95	0
9	51.39714	42.91	5.521429	0.171429
10	51.26	43.07	5.35	0.32
11	51.32857	42.85	5.435714	0.385714
12	51.58	43.07	5.35	0
13	51.58	42.79	5.63	0
14	51.58	42.79	5.35	0.28
15	51.32857	42.99	5.595714	0.085714

2.2.3 Production of LDPE- bituminous mixtures

LDPE-modified bituminous concrete specimens were fabricated following the Marshall mix design method (ASTM D6927, 2015) and compacted using 50 blows per face with a compaction hammer dropped from a height of 450 mm, suitable for medium traffic loads. For sample preparation, the aggregates, measured according to the specified mix design (see Table 3), were heated to 180°C, while the bitumen and LDPE were heated together to 140°C. These components

were then combined and mixed thoroughly at 180°C until a uniform blend was formed. The resulting mixture was poured into preheated molds and compacted with 50 blows on each side at temperatures between 140°C and 150°C. Before testing, the specimens were preheated to 60°C and then subjected to moisture resistance evaluation using a Marshall testing machine at a constant strain rate of 50.8 mm/min. A cross-sectional view of some of the prepared LDPE-modified bituminous concrete specimens is shown in Plate 1.



Plate 1: LDPE- Bituminous Concrete Samples

2.2.4 Brittleness Ratio Measure of LDPE- bituminous mixtures

The brittleness of prepared bituminous concrete samples was measured as the strength ratio (S_R) determined as the ratio of compressive strength to tensile strength (Equation 5).

$$S_R = \frac{\sigma_C}{\sigma_T} \quad (5)$$

Where; S_R = brittleness ratio or strength ratio

σ_C = compressive strength of LDPE modified bituminous samples

σ_T = tensile strength of LDPE modified bituminous samples

2.2.4 Mixture Regression Model

2.2.4.1 Model Concept

The general second order (q, m) mixture regression model has a general form represented by Equation (6);

$$Y = b_0 + \sum b_i z_i + \sum b_{ij} z_i z_j + \sum b_{ijk} z_i z_j z_k + \dots + \sum b_{i_1, i_2, \dots, i_m} z_{i_1} z_{i_2} z_{i_m} \quad (6)$$

Where; $1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq j \leq k \leq q$

b_0 is a constant coefficient

For (4, 2) mixture or quadratic problem as adopted in this study, the general mixture regression model, Equation (6) becomes;

$$Y = b_0 + b_1 Z_1 + b_2 Z_2 + b_3 Z_3 + b_4 Z_4 + b_{12} Z_1 Z_2 + b_{13} Z_1 Z_3 + b_{14} Z_1 Z_4 + b_{23} Z_2 Z_3 + b_{24} Z_2 Z_4 + b_{11} Z_1^2 + b_{22} Z_2^2 + b_{33} Z_3^2 + b_{44} Z_4^2 \quad (7)$$

For a quaternary mixture, Equation (8) is obtained;

$$Z_1 + Z_2 + Z_3 + Z_4 = 1 \quad (8)$$

Multiplying through by constant b_0 , yields Equation (9).

$$b_0 Z_1 + b_0 Z_2 + b_0 Z_3 + b_0 Z_4 = b_0 \quad (9)$$

Again, multiplying Equation (8) by Z_1, Z_2, Z_3 , and Z_4 in succession and rearranging, Equation (10) is produced.

$$\begin{cases} Z_1^2 = Z_1 - Z_1 Z_2 - Z_1 Z_3 - Z_1 Z_4 \\ Z_2^2 = Z_2 - Z_1 Z_2 - Z_2 Z_3 - Z_2 Z_4 \\ Z_3^2 = Z_3 - Z_1 Z_3 - Z_2 Z_3 - Z_3 Z_4 \\ Z_4^2 = Z_4 - Z_1 Z_4 - Z_2 Z_4 - Z_3 Z_4 \end{cases} \quad (10)$$

Substituting Equations (10) and (9) into Equation (7), Equation (11) was obtained after necessary transformation.

$$Y = (b_0 + b_1 + b_{11}) Z_1 + (b_0 + b_2 + b_{22}) Z_2 + (b_0 + b_3 + b_{33}) Z_3 + (b_0 + b_4 + b_{44}) Z_4 + (b_{12} - b_{11} - b_{22}) Z_1 Z_2 + (b_{13} - b_{11} - b_{33}) Z_1 Z_3 + (b_{14} - b_{11} - b_{44}) Z_1 Z_4 + (b_{23} - b_{22} - b_{33}) Z_2 Z_3 + (b_{24} - b_{22} - b_{44}) Z_2 Z_4 + (b_{34} - b_{33} - b_{44}) Z_3 Z_4 \quad (11)$$

Denoting; $\beta_i = b_0 + b_i + b_{ii}$ and

$$\beta_{ij} = b_{ij} - b_{ii} - b_{jj}$$

The reduced second-degree mixture regression model in 4 variables is shown by Equation (12).

$$Y = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_4 Z_4 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{14} Z_1 Z_4 + \beta_{23} Z_2 Z_3 + \beta_{24} Z_2 Z_4 + \beta_{34} Z_3 Z_4 \quad (12)$$

The number of coefficients has reduced from 14 in Equation (7) to 10 in Equation (12). Thus, the reduced second-degree polynomial in q-variables is as shown by Equation (13).

$$Y = \sum_{1 \leq i \leq q} \beta_i Z_i + \sum_{i \leq j \leq q} \beta_{ij} Z_i Z_j$$

Where;

Y = Expected response

β_i, β_{ij} = Coefficients of the mixture quadratic regression model

Z_i, Z_j = Proportion of LDPE modified bituminous mixtures components

2.2.4.2 Mixture Regression Model Calibration

In a short mathematical or matrix form, Equation (12) becomes;

$$Y = [Z_i][\beta_i] \quad (13)$$

$[Z_i]$ = shape function vector showing interaction between components

$[\beta_i]$ = model coefficient vector function

Y = vector of Brittleness ratio or strength ratio (S_R)

The coefficient function is represented by Equation (14)

$$[\beta_i] = [\beta_1, \beta_2, \beta_3, \beta_4, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{23}, \beta_{24}, \beta_{34},]^T \quad (14)$$

The coefficient vector or column matrix (Equation, 14) can be determined from least square estimator given by Equation (15)

$$[\beta_i] = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \beta_{23} \\ \beta_{24} \\ \beta_{34} \end{bmatrix} = (Z^T Z)^{-1} Z^T Y \quad (15)$$

Where; Z^T = transpose of the shape function or transpose of the matrix of predictor variables.

The special consideration here is that Equation (15) is subjected to the constraints shown in Equation (8), hence, a case of a constrained mixture regression model suffices. Consequently, the coefficients in the constrained mixture regression model (Equation 3.33) were estimated or determined using least square method via Microsoft excel.

2.2.4.2 Mixture Regression Model Calibration

Developed mixture regression model was subjected to F-distribution test and mean absolute percentage difference (MAP.D) statistics for validation.

F-distribution test; The F-statistics is given as the ratio of variance (Equation 16) between the predicted/model response value and that of experimental or observed value.

$$F = \frac{S_1^2}{S_2^2} \quad (16)$$

Where; S_1^2 = Larger of both variances

S_2^2 = Smaller of both variances

S^2 is obtained from Equation (17)

$$S^2 = \frac{1}{n-1} [\sum(Y - \bar{Y})^2] \quad (17)$$

Where: \bar{Y} = Average mean of response, Y

Y = Means of response

The following hypothesis were adopted;

Null Hypothesis: H_0 = there is no significant difference between the experimental and model responses.

Alternate Hypothesis: H_1 = there is a significant difference between the experimental and model responses.

The null hypothesis is accepted when the F-value calculated in accordance to Equation (16) is less than tabulated value (from F-distribution table) and the model is declared adequate. Otherwise, the alternate hypothesis is accepted and the model is considered inadequate.

MAP.D Statistics; MAP.D simply quantifies the average relative difference between predicted or measured values and actual values in percentage terms. It is commonly used in forecasting, regression analysis, and comparing model accuracy. Equation (18) was used for the determination of MAP.D.

$$MAP.D = \frac{1}{n} \sum \left(\frac{\text{Experimental response} - \text{model response}}{\text{Average of both responses}} \times 100 \right) \quad (18)$$

Where; n represents the number of data sets.

3. RESULTS AND DISCUSSION

3.1 Brittleness Ratio Results of LDPE-modified Bituminous Concrete

Figure 1 illustrates the brittleness ratio results for the LDPE-modified bituminous concrete mixtures, with values ranging from 1.998 at run 2 to 3.169 at run 6. This variation highlights a significant increase in stiffness due to the addition of LDPE. The enhancement of stiffness in LDPE-modified binders is well documented in recent research. Singh and Gupta (2024) demonstrated that incorporating LDPE into asphalt

mixtures improves mechanical properties such as penetration, softening point, and viscosity, which collectively contribute to increased resistance to permanent deformation and rutting. Such improvements underscore the capacity of LDPE to enhance the structural performance of asphalt under heavy traffic loading.

However, the brittleness ratio exceeding 3.0, as observed in run 6, raises concerns about the potential for increased brittleness, especially under low-temperature conditions. Ullah, Singh, and Gupta (2024) found that

while higher LDPE concentrations increase stiffness, they also lead to reduced ductility and greater susceptibility to cracking at lower temperatures. This brittleness is a critical factor because it can undermine the durability of pavements in colder climates or during temperature fluctuations, which stresses the importance of optimizing LDPE dosage within the mix design to strike a balance between stiffness and flexibility.

Therefore, the findings suggest that although LDPE modification effectively enhances the stiffness and rutting resistance of bituminous mixtures, careful attention must be paid to the brittleness ratio. Maintaining this ratio below the critical threshold of 3.0 can help prevent premature cracking while still benefiting from the mechanical enhancements provided by LDPE. Mix designs should consider these trade-offs and may require complementary modifiers or performance tests such as Dynamic Modulus assessments, to ensure comprehensive evaluation of mixture behaviour across temperature ranges and loading conditions.

To achieve a balance between enhanced stiffness and adequate ductility, it is recommended to incorporate LDPE into asphalt mixtures at an optimal dosage, offering improved resistance to permanent deformation without significantly compromising ductility. Exceeding the optimal dosage can lead to increased brittleness, as indicated by higher brittleness ratios, which may adversely affect the material's performance under low-temperature conditions. Therefore, careful consideration and precise control of LDPE content are essential to ensure the desired balance between stiffness and ductility in modified asphalt mixtures.

In summary, the inclusion of LDPE in bituminous concrete mixtures offers promising improvements in pavement stiffness and deformation resistance, aligning with sustainable construction goals. Yet, controlling brittleness remains essential to achieve a durable, high-performing pavement. This balance is vital for maximizing the benefits of LDPE-modified asphalt while minimizing the risks associated with increased brittleness at higher additive concentrations.

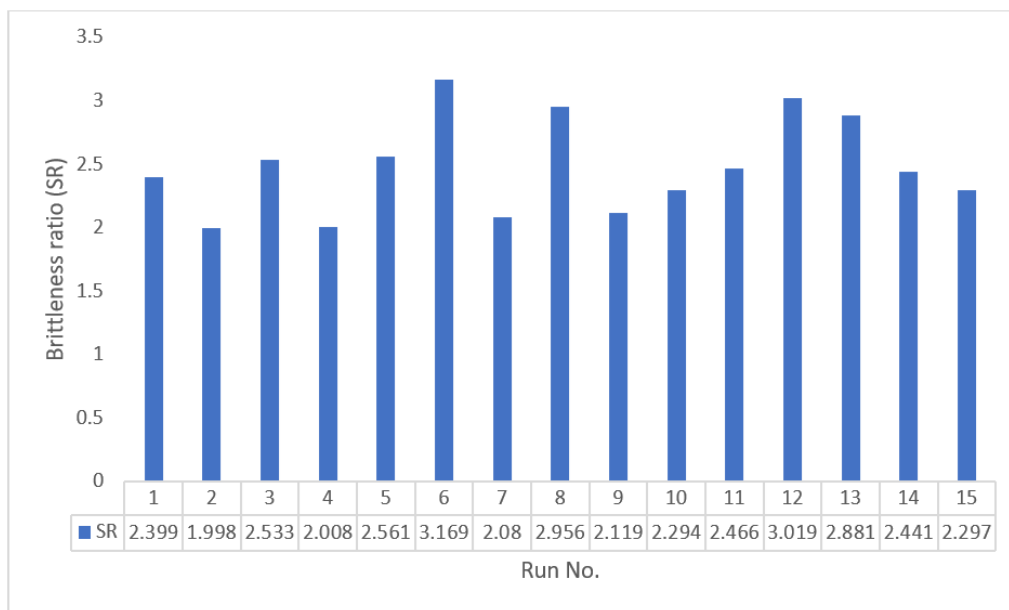


Figure 1: Brittleness Ratio of LDPE-Bituminous Concrete Mixtures

3.2 Mixture Regression Model for Predicting Brittleness of LDPE-Bituminous Concrete Mixtures

Table 4 presents the shape vector, Z and the experimentally determined S_R of LDPE-modified

bituminous concrete samples. β or coefficient matrix as presented in Equation (19) was obtained by the application of Equation (15) using Microsoft excel with details presented in Figure 2.

$$[\beta_i] = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \beta_{23} \\ \beta_{24} \\ \beta_{34} \end{bmatrix} = \begin{bmatrix} 761.9523 \\ 742.9327 \\ 908.3908 \\ 1072.853 \\ -30.2452 \\ -40.0024 \\ -39.8194 \\ 8.998726 \\ 4.085244 \\ 6.738235 \end{bmatrix} \quad (15)$$

By substituting these coefficient values into Equation (12), the mixture regression model for predicting the brittleness ratio, S_R of LDPE-modified bituminous concrete was formulated, as presented in Equation (16).

$$SR_{LDPE-mBC} = 761.9523Z_1 + 742.9327Z_2 + 908.3908Z_3 + 1072.853Z_4 - 30.2452Z_1Z_2 - 40.0024Z_1Z_3 - 39.8194Z_1Z_4 + 8.998726Z_2Z_3 + 4.085244Z_2Z_4 + 6.738235Z_3Z_4 \quad (16)$$

Where; Z_1 represents the proportion of granite in LDPE-modified bituminous concrete
 Z_2 represents the proportion of sand in LDPE-modified bituminous concrete
 Z_3 represents the proportion of bitumen in LDPE-modified bituminous concrete
 Z_4 represents the proportion of LDPE plastics in LDPE-modified bituminous concrete

Table 4: Shape Matrix or Vector of Components and Brittleness Ratio, S_R for LDPE-Modified Bituminous Concrete

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4	$Y = S_R$
51.489	42.850	5.436	0.226	2206.285	279.8772	11.62171	232.9204	9.671857	1.226918	2.399
51.489	42.990	5.436	0.086	2213.494	279.8772	4.413306	233.6814	3.684857	0.465918	1.998
51.329	42.990	5.436	0.246	2206.615	279.0074	12.61216	233.6814	10.56326	1.335633	2.533
51.489	42.850	5.576	0.086	2206.285	287.0856	4.413306	238.9194	3.672857	0.477918	2.008
51.260	42.790	5.350	0.600	2193.415	274.241	30.756	228.9265	25.674	3.21	2.561
51.260	43.070	5.670	0.000	2207.768	290.6442	0	244.2069	0	0	3.169
51.329	42.850	5.736	0.086	2199.429	294.406	4.399592	245.7754	3.672857	0.491633	2.080
51.260	42.790	5.950	0.000	2193.415	304.997	0	254.6005	0	0	2.956
51.397	42.910	5.521	0.171	2205.451	283.7857	8.810939	236.9245	7.356	0.946531	2.119
51.260	43.070	5.350	0.320	2207.768	274.241	16.4032	230.4245	13.7824	1.712	2.294
51.329	42.850	5.436	0.386	2199.429	279.0074	19.79816	232.9204	16.52786	2.096633	2.466
51.580	43.070	5.350	0.000	2221.551	275.953	0	230.4245	0	0	3.019
51.580	42.790	5.630	0.000	2207.108	290.3954	0	240.9077	0	0	2.881
51.580	42.790	5.350	0.280	2207.108	275.953	14.4424	228.9265	11.9812	1.498	2.441
51.329	42.990	5.596	0.086	2206.615	287.22	4.399592	240.5598	3.684857	0.479633	2.297

Where; Z_1 = granite proportion; Z_2 = sand proportion, Z_3 = bitumen proportion; Z_4 = LDPE proportion

3.2.1 S_R Model Validation for LDPE-Modified Bituminous Concrete

Table 5 summarizes the F-distribution test results used to validate the regression model developed for predicting the brittleness ratio (S_R) of LDPE-modified bituminous concrete mixtures. The computed F-value of 1.382 is substantially lower than the critical F-value of 2.48, corresponding to a 5% significance level and 14 degrees of freedom. This statistical comparison supports the acceptance of the null hypothesis, indicating that the regression model sufficiently fits the experimental data and is adequate for predicting the S_R of the LDPE-modified mixtures. Such an approach to model validation aligns with standard practices in regression analysis, where a lower F-calculated relative to F-critical suggests the model does not significantly

deviate from observed data variability (Montgomery *et al.*, 2012).

Complementing this finding, Figure 6 illustrates the coefficient of determination (R^2) for the S_R model, which was calculated as 70.38%. An R^2 of over 70% implies that the model explains a significant portion of variance within the dataset, indicating strong predictive capability. In material and pavement engineering studies, an R^2 exceeding 0.7 is generally considered indicative of a reliable model capable of capturing the essential behaviour of the system under study (Hosmer *et al.*, 2013). This level of explained variance reflects the model's robustness in capturing the influence of mixture components on the S_R , thereby enhancing confidence in the model's use for design and optimization.

Furthermore, the Mean Absolute Percentage Deviation (MAP.D) analysis presented in Table 6 reinforces the model's predictive accuracy by quantifying the average relative error between predicted and experimental SR values. The reported MAP.D value of 7.2% is sufficiently low to conclude that the predicted strength ratios are closely aligned with the actual measurements. This low deviation is consistent with accepted engineering tolerances and demonstrates the practical applicability of the model in estimating SR

within an acceptable margin of error (Willmott & Matsuura, 2005).

Together, the F-test results, R² statistic, and MAP.D analysis provide a comprehensive validation framework, supporting the credibility and practical utility of the developed regression model for LDPE-modified bituminous concrete. This multipronged validation approach is critical, as relying solely on R² can be misleading due to potential overfitting, while the F-test and MAP.D provide insights into model significance and prediction accuracy respectively (James *et al.*, 2013).

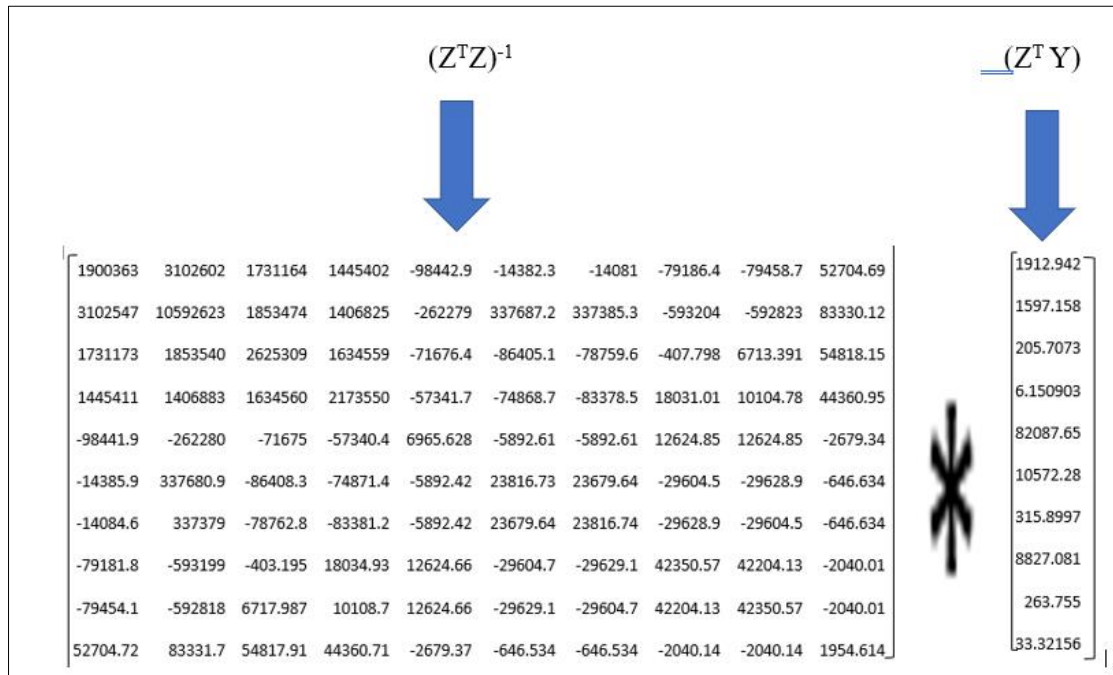


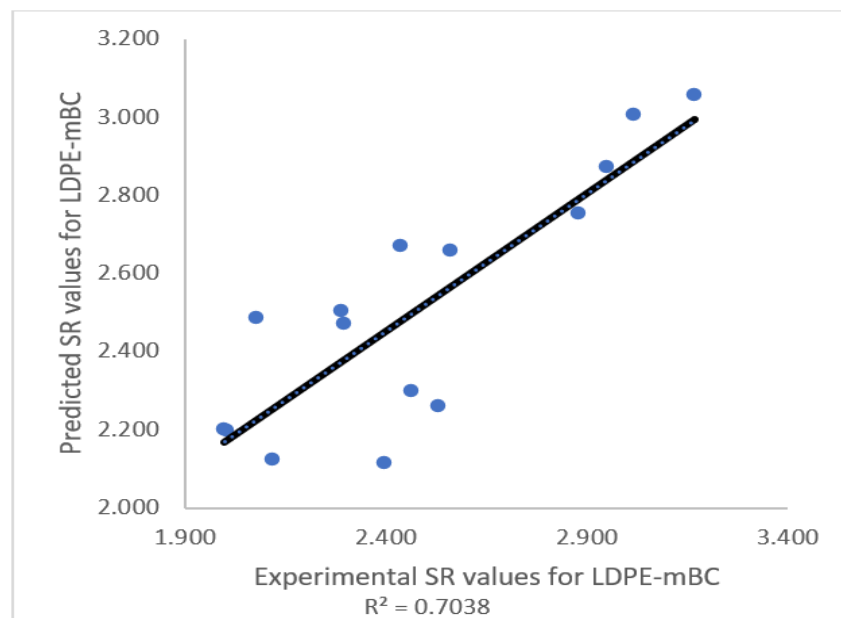
Figure 2: Coefficient Determination for Brittleness Ratio, SR Model of LDPE-Modified Bituminous Concrete

Table 5: F-Distribution Statistics for Validation of SR Model of LDPE-Modified Bituminous Concrete

Experimental SR Values= Y_e	Predicted SR Value= Y_p	$Y_e - \hat{Y}_e$	$Y_p - \hat{Y}_p$	$(Y_e - \hat{Y}_e)^2$	$(Y_p - \hat{Y}_p)^2$
2.399	2.111	-0.025	-0.329	0.00065	0.10802
1.998	2.199	-0.427	-0.241	0.18232	0.05806
2.533	2.259	0.108	-0.181	0.01167	0.03272
2.008	2.196	-0.416	-0.243	0.17337	0.05928
2.561	2.658	0.136	0.218	0.01858	0.04755
3.169	3.057	0.744	0.617	0.55372	0.38082
2.080	2.485	-0.345	0.046	0.11911	0.00209
2.956	2.871	0.531	0.431	0.28188	0.18610
2.119	2.121	-0.305	-0.318	0.09328	0.10130
2.294	2.503	-0.130	0.063	0.01698	0.00398
2.466	2.294	0.041	-0.146	0.00169	0.02121
3.019	3.004	0.594	0.565	0.35267	0.31873
2.881	2.751	0.456	0.311	0.20826	0.09665
2.441	2.667	0.016	0.228	0.00025	0.05186
2.297	2.471	-0.128	0.031	0.01626	0.00097
$\hat{Y}_e = 2.425$	$\hat{Y}_p = 2.440$			$\Sigma = 2.03068$	$\Sigma = 1.46934$
Square of deviation of experimental SR values from mean SR value			S_e^2	0.145048812	
Square of deviation of predicted SR values from mean SR value			S_p^2	0.10495303	
F- Calculated value, ratio of the two deviations (Equation 3.34), F-cal				1.382035552	

Table 6: MAP.D Statistics for Comparing Model to Experimental S_R Values of LDPE-Modified Bituminous

Experimental Values= Y_e	SR	Predicted SR Value= Y_p	ABS ($Y_e - Y_p$) = B	($Y_e + Y_p$)/2 = C	B/C	(B/C)*100
2.399		2.111	0.288	2.255	0.12779	12.77896
1.998		2.199	0.201	2.098	0.09581	9.58115
2.533		2.259	0.274	2.396	0.11434	11.43407
2.008		2.196	0.188	2.102	0.08939	8.93853
2.561		2.658	0.097	2.609	0.03708	3.70795
3.169		3.057	0.112	3.113	0.03598	3.59841
2.080		2.485	0.406	2.282	0.17780	17.78004
2.956		2.871	0.085	2.913	0.02901	2.90127
2.119		2.121	0.002	2.120	0.00101	0.10135
2.294		2.503	0.208	2.399	0.08688	8.68842
2.466		2.294	0.172	2.380	0.07214	7.21429
3.019		3.004	0.014	3.011	0.00475	0.47475
2.881		2.751	0.130	2.816	0.04633	4.63345
2.441		2.667	0.227	2.554	0.08883	8.88255
2.297		2.471	0.174	2.384	0.07287	7.28725
Absolute percentage difference (AP. D)						108.00244
Mean Absolute percentage difference (MAP.D)						7.20016

**Figure 6: R^2 Statistics for S_R Model of LDPE modified Bituminous Concrete**

4. CONCLUSIONS

From the results and discussion of this study, these insightful conclusions are hereby put forward;

- I. The brittleness ratio of LDPE-modified bituminous concrete ranged from 1.998 to 3.169, confirming that LDPE significantly increases stiffness; however, brittleness ratios above 3.0 indicate a risk of brittleness that could impair pavement durability, highlighting the need to optimize LDPE dosage.
- II. The regression model predicting brittleness ratio from granite, sand, bitumen, and LDPE proportions showed strong validity, with an F-calculated value of 1.382 below the critical 2.48 and an R^2 of 70.38%, indicating it reliably explains over 70% of the brittleness variance.

- III. A Mean Absolute Percentage Deviation of 7.2% between predicted and experimental values confirms the model's accuracy within acceptable engineering tolerance, and combined validation metrics (F-test, R^2 , MAP.D) provide a robust assessment of model performance.
- IV. As deduced, while LDPE modification improves stiffness, it necessitates precise mixture design to manage brittleness. The validated regression model provides an effective means to fine-tune mixture components, achieving a balance between enhanced performance and durability for sustainable, high-performance pavements.

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