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Teaching "Integration by Parts" in Calculus 12: A Pedagogical Experiment Based on the Inductive Instruction

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Abstract: Inductive reasoning, or induction, makes generalizations from particular facts or instances. Based on induction, in teaching, it is developed to become inductive instruction. With this approach, the learning process of students will begin with investigation of specific instances to draw general conclusions. In this study, we applied the inductive instruction to guide students how to use the formula "integration by parts" in Calculus 12 to compute integrals. The results showed that students were active and they knew how to develop strategies for applying the formula "integration by parts" in an effective way.
Keywords: Induction, inductive instruction, teaching calculus, integration by parts, mathematics education.

THEORETICAL BACKGROUND

Induction

Inductive reasoning, or induction, makes generalizations from particular facts or instances. It is contrasted with deduction, the reasoning process which begins with a general to reach a specific, logical conclusion.

Inductive instruction

Inductive instruction could be described as follows:

The instruction begins with specifics - a set of observations or experimental data to interpret, a case study to analyze, or a complex real-world problem to solve. As the students attempt to analyze the data or scenario or solve the problem, they generate a need for facts, rules, procedures, and guiding principles, at which point they are either presented with the needed information or helped to discover it for themselves [2].

The role of induction in instruction

According to Nguyen Canh Toan, induction plays a major role in fostering intelligence for students, and he pointed out that: "Teaching mathematics just for the purpose of transmission of knowledge will lead to take deduction seriously than induction. But if focusing on "training creative intelligence" for students, the role of induction will be on par with the one of deduction" [3].

STATEMENT OF RESEARCH PROBLEM

In mathematics curriculum for secondary schools of Vietnam, topic "Primitive and Integration" (Nguyên hàm và Tích phân) is in Calculus 12 (Giải tích 12) [1]. About methods for calculating integration, the textbook "Calculus 12" introduced method of change of variables and method of integration by parts. For integration by parts, the textbook presented the following theorem:

THEOREM:
If u = u(x) and v = v(x) are two functions which have continuous derivatives on the closed interval [a; b], then

$$\int_a^b u(x)v'(x)dx = u(x)v(x)\Big|_a^b - \int_a^b u'(x)v(x)dx$$

or

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu .$$

About types of problem in Calculus 12 for applying method of integration in parts, the textbook provides the following types:

The first type of problem (T1): Compute $\int_a^b P(x)e^{kx}dx$, where P(x) is polynomial;

The second type of problem (T2): Compute $\int_a^b P(x)\cos kxdx$ and $\int_a^b P(x)\sin kxdx$, where P(x) is polynomial;

The third type of problem (T3): Compute $\int_a^b P(x)\ln^k xdx$, where P(x) is polynomial;

The fourth type of problem (T4): Compute $\int_a^b e^{lx}\cos kxdx$ and $\int_a^b e^{lx}\sin kxdx$.

The problem is that how to help the students solve any exercise of the above types of problem. In the study, we chose an inductive instruction to enhance active activities of students in discovering general principles for solving the four types of problem mentioned as above through solving particular exercises.

METHODOLOGY

In order to evaluate effectiveness of the inductive instruction, we compared learning outcomes of the students of two experimental classes with the ones of a control class in which deductive instruction was used(in academic year: 2015 -2016).

Experimental classes: 12A2 and 12A3 - The senior secondary school "Quốc Thái", An Phú district, An Giang province, Vietnam (see Table 1).
Control class: 12A6 -The senior secondary school "Quốc Thái", An Phú district, An Giang province, Vietnam (see Table 1).

Table 1: Information about experimental classes and control class

Experimental Class	12A2 (27 students)	12A3 (28 students)
Control class	12A6 (38 students)	

The process of teaching each type of problem in each experimental class

The process of teaching was designed based on the inductive instruction; this process consists of 4 phases as follows (see Table 2):
Phase 1: Give an exercise of the above types of problem to students and ask them for creating analogous exercises.
Phase 2: Choose one of the exercises which were created by the students. The students work in group of four to find out strategies to apply "method of integration by parts" for solving the exercise chosen, and choose the best strategy.
Phase 3: Ask students for presenting the solution of the exercise according to the strategy which their group has just chosen in the blackboard
Phase 4: Ask students for generalizing the exercise with the corresponding solving strategy.

Table 2: The Objectives and students' activities in each phase of experimental teaching

Phase	Objective	Activity of students
1	Help Students identify features of a sample exercise	Observe and give some similar exercises individually
2	Help students find out how to apply method of integration by parts to solve one exercise suggested in Phase 1	Work in group to find out strategies for applying method of integration in parts to solve the exercise and choose the best strategy
3	Identify students' ability to apply method of integration by parts to solve the chosen exercise	Present the solution of the chosen exercise
4	Help students generalize the exercise	Work in group to state the general form of the exercise.

The instruction in control class

In control class, the teacher of the senior secondary school used traditional method to give lesson: Firstly, the teacher reviewed method of integration in parts, and then he gave his students four exercises to apply this method.

1) A = $\int_0^{\frac{\pi}{2}} (x+1)\cos x dx$; 2) B = $\int_0^1 2xe^{2x} dx$; 3) C= $\int_1^2 2x\ln x dx$; 4) D = $\int_0^1 (x-3)e^x dx$

Time for teaching: 50 minutes for practicing to apply the formula "integration by parts" in both experimental classes and control class

RESULTS AND DISCUSSION

- ***The process of experimental teaching and outcomes***

For teaching T1: Table 3 presented the process of teaching T1 in two experimental classes and the outcomes of students in each phase.

Table 3: The outcomes and process of teaching T1

Phase	**Objective**	**Activity of students**	**Outcomes**	
			12A2	**12A3**
1	Help Students identify features of $\int_0^1 xe^x dx$	Observe and give some similar exercises individually	$\int_0^1 x^2e^x dx$, $\int_0^1 xe^{2x} dx$, $\int_0^1 (x+1)e^{2x} dx$, $\int_0^1 (x^2+1)e^x dx$.	$\int_0^1 x^2e^{2x} dx$, $\int_0^1 (x+2)e^x dx$, $\int_0^1 (x^2+2x+1)e^{x^2} dx$, $\int_0^1 (x+1)e^{2x} dx$, $\int_0^1 (x+1)e^x dx$.
2	Help students find out how to apply method of integration by parts to solve one exercise suggested in Phase 1: Compute: $I = \int_0^1 xe^x dx$	Work in group to find strategies for applying method of integration in parts to solve the exercise and choosing the best strategy	Strategies suggested: 1. $\begin{cases} u = x \\ dv = e^x dx \end{cases}$ 2. $\begin{cases} u = e^x \\ dv = xdx \end{cases}$ 3. $\begin{cases} u = xe^x \\ dv = dx \end{cases}$ The best strategy: 1	Strategies suggested 1. $\begin{cases} u = x \\ dv = e^x dx \end{cases}$ 2. $\begin{cases} u = x \\ dv = e^x dx \end{cases}$ The best strategy: 1
3	Identify students' ability to apply method of integration by parts to compute I	Present the solution of the chosen exercise	Let $\begin{cases} u = x \\ dv = e^x dx \end{cases}$, I = 1 (True)	Let $\begin{cases} u = x \\ dv = e^x dx \end{cases}$, I = 1 (True)
4	Help students generalize the exercise	Work in group to state the general form of the exercise.	$\int_a^b (ax^2+bx+c)e^x dx$ $\int_a^b (ax+b)e^x dx$	$\int_a^b (ax^2+bx+c)e^x dx$ $\int_a^b (ax+b)e^x dx$

			$\int_a^b (ax^2+bx+c)e^{2x}dx$ $\int_a^b (ax^2+bx+c)e^{kx}dx$	$\int_a^b (ax^2+bx+c)e^{2x}dx$ $\int_a^b (ax^2+bx+c)e^{x^2}dx$

After the above phases in each experimental class, the teacher summarized as follows:

If $\int_a^b P(x)e^{kx}dx$, where P(x) is polynomial, then we will let u = P(x),

and dv = e^{kx}dx, and apply the method of integration by parts to solve.

For teaching T2: Table 4 showed the outcomes of students and process of teaching T2

Table 4: The outcomes and process of teaching T2

Phase	**Objective**	**Activity of students**	**Outcomes**	
			12A2	**12A3**
1	Help Students identify features of the exercise: $\int_0^1 x\cos xdx$	Observe and give some similar exercises individually	$\int_0^{\frac{\pi}{2}} (x+1)\cos xdx$ $\int_0^{\frac{\pi}{2}} x^2\cos xdx$ $\int_0^{\frac{\pi}{2}} x\cos 2xdx$ $\int_0^{\frac{\pi}{2}} x\cos 3xdx$ $\int_0^{\frac{\pi}{2}} (x+1)\sin xdx$ $\int_0^{\frac{\pi}{2}} x^2\sin xdx$ $\int_0^{\frac{\pi}{2}} x\sin 2xdx$ $\int_0^{\frac{\pi}{2}} x\sin 3xdx$	$\int_0^{\frac{\pi}{2}} (x+1)\cos xdx$ $\int_0^{\frac{\pi}{2}} (x^2+1)\cos xdx$ $\int_0^{\frac{\pi}{2}} (x+1)\cos 2xdx$ $\int_0^{\frac{\pi}{2}} (x+1)\cos 3xdx$ $\int_0^{\frac{\pi}{2}} (x+1)\sin xdx$ $\int_0^{\frac{\pi}{2}} (x^2+1)\sin xdx$ $\int_0^{\frac{\pi}{2}} (x+1)\sin 2xdx$ $\int_0^{\frac{\pi}{2}} x\sin xdx$
2	Help students find out how to apply method of integration by parts to solve one exercise suggested in Phase 1: Compute	Work in group to find strategies for applying method of integration in parts to solve the exercise and choosing the best strategy	Strategies suggested: 1. $\begin{cases} u = x \\ dv = \cos xdx \end{cases}$	Strategies suggested: 1. $\begin{cases} u = x \\ dv = \cos xdx \end{cases}$

	J= $\int_0^1 x\cos x dx$		2. $\begin{cases} u = \cos x \\ dv = xdx \end{cases}$ The best strategy: 1	2. $\begin{cases} u = \cos x \\ dv = xdx \end{cases}$ The best strategy: 1
3	Identify students' ability to apply method of integration by parts to solve the chosen exercise	Present the solution of the chosen exercise	Let $\begin{cases} u = x \\ dv = \cos xdx \end{cases}$, J= $\frac{\pi}{2} - 1$.	Let $\begin{cases} u = x \\ dv = \cos xdx \end{cases}$, J= $\frac{\pi}{2} - 1$.
4	Help students generalize the exercise	Work in group to state the general form of the exercise.	$\int_a^b (ax^2 + bx + c)\cos xdx$ $\int_a^b (ax + b)\cos xdx$ $\int_a^b (ax^2 + bx + c)\cos kxdx$ $\int_a^b x\cos kxdx$	$\int_a^b (ax^2 + bx + c)\cos 2xdx$ $\int_a^b (ax + b)\cos 2xdx$ $\int_a^b (ax^2 + bx + c)\cos x^2 dx$ $\int_a^b x\cos kxdx$

After the above phases in each experimental class, the teacher summarized as follows:

1. If $\int_a^b P(x)\cos kxdx$, where P(x) is polynomial, then we will let u = P(x) and dv = coskxdx, and apply the method of integration by parts to solve.
2. If $\int_a^b P(x)\sin kxdx$, where P(x) is polynomial, then we will let u = P(x) and dv = sinkxdx, and apply the method of integration by parts to solve.

For teaching T3 and T4, the instruction was implemented according to the process similar to the one applied for teaching T1 and T2.

- ***Test of the achievement of students after giving lesson***

In order to evaluate the effectiveness of the use of inductive instruction into teaching, at the end of each lesson of 12 A2, 12 A3 and 12A6, we tested the achievement of students by following exercises:

"Compute the following integrals

1. I= $\int_0^1 (x+1)e^x dx$ 2. J= $\int_0^{\frac{\pi}{2}} (x+1)\sin 2xdx$; 3. H= $\int_0^1 x^2 \ln e^x dx$; 4) K= $\int_0^{\frac{\pi}{2}} e^x \sin xdx$".

Table 5: Comparing the students' results of solving exercises

Class	**Compute I**		**Compute J**		**Compute H**		**Compute K**	
	True Solution	(%)	True Solution	(%)	True Solution	(%)	True Solution	(%)
12A2 (n=27)	27	100%	27	100%	27	100%	22	81,48%
12A3 (n=28)	28	100%	28	100%	28	100%	27	96,43%
12A6 (n=38)	29	76,32%	9	23,68%	28	73,68%	0	0%

Table 5 showed that all students of two experimental classes (12A2, 12A3) computed I, J, H exactly; while many students of control class (12A6) gave wrong solutions to I, J, H and especially no students of this class could compute K.

CONCLUSION

From the pedagogical experiment, we could conclude that teaching mathematics by inductive instruction could offer students opportunities to activate thinking activities in learning process; and their outcomes could be good Therefore, teaching mathematics with inductive instruction could help students not only enhance their knowledge of mathematics, but be acquainted with creative activities in mathematics.

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