"Didactical Contract" As a Tool for Finding out Students' Errors in Solving Problem: An Illustration in Analytic Geometry

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Abstract: In this study, we used the concept "didactical contract", which was introduced by Guy Brousseau in 1980, as a tool for finding out the errors of students. In order to illustrate this approach, we used the type of a task: "In the space Oxyz, write the general equation of a plane (P) passing through point $A(x_0, y_0, z_0)$ and parallel to two straight lines: (d_1) and (d_2) ((d_1) and (d_2) are not parallel to each other)" which was mentioned in mathematics textbooks of Vietnam. We formulated the hypothesis H: there exists a rule of didactical contract: When solving the problem, students don't verify whether (d_1) and (d_2) are parallel or not; therefore, students will commit errors in the case of (d_1) and (d_2) are parallel. The results of study showed that many students committed errors in solving the exercise because of the above didactical contract.

Keywords: Didactical contract, error in solving problem, analytic geometry, mathematics education.

INTRODUCTION

In process of teaching mathematics, how do recognize students' error is critical to the teacher, because from which he adjusts his instruction to help his students understand knowledge in a correct way and prevent errors that his students could commit. In this study we used the concept "didactical contract", which was introduced by Guy Brousseau in 1980, as a tool for finding out the errors of students [1].

THEORETICAL BACKGROUND
Didactical Contract

The concept "didactical contract" was described by Guy Brousseau in 1980 as follows [1]:

"Students tend to make any information or limitation clear using what the teacher, whether consciously or unconsciously, produces in his teaching activity. We think about the most common habits in teaching, and we define a didactical contract as the specific behavior that students expect from teachers and teachers expect from students too".

The didactical contract gives rules involving expectations and behavior of students and teachers towards knowledge. It points to what students and teacher have to do, their roles and their responsibilities one to another, in an implicit way.

Error in solving problem

Error in solving problem is an error caused by improperly implementing mathematical rules; by applying the incorrect mathematical formulas, mathematical theorems; or by misunderstanding concepts, theorems; by misunderstanding an assignment, or by making mistake in calculation and presenting problem solution [2, 3].

Error analysis

In teaching process**,** according to Marzano [4], error analysis is one of activities for extending and refining knowledge; analyzing error is to find out the the answer to the following questions:

"What are the errors in reasoning in this information?
How is this information misleading?
How could it be corrected or improved" [4]

Statement of research problem and research objectives

In process of learning analytic geometry in space, secondary school students of Vietnam learn how to solve many types of task relating to write the equation of a plane with given conditions. In this study, we were concerning about the type of task as follows: "In the space Oxyz , write the equation of a plane (P) through point A(x_0, y_0,z_0) and parallel to two straight lines (d$_1$) and (d$_2$)" (E). In textbooks and exercise books (d$_1$) and (d$_2$) in this problem intersect each other, or they are skew lines.

For (E), students can use the following strategy to solve:

The strategy (S):

- Step 1: Find coordinates of $\vec{u}_1$ (direction vector of d$_1$) and $\vec{u}_2$ (direction vector of d$_2$)

    Choose vector pair ($\vec{u}_1$ $\vec{u}_2$), to be a vector pair of direction of plane (P);

- Step 2: Find the coordinates of normal vector of plane (P): $\vec{n} = \left[\vec{u}_1; \vec{u}_2\right]$;

- Step 3: The equation of plane (P): ax + by + cz + d = 0 was determined due to the coordinates of $\vec{n}$ and point A.

From (E) and (S), we have hypothesis as follows:

H: For solving (E), there exists a rule of didactical contract: Students don't verify whether (d$_1$) and (d$_2$) are parallel or not; therefore, students will commit errors if (d$_1$) and (d$_2$) are parallel,

METHODOLOGY

Problem used to verify the hypothesis

In order to verify the above two hypotheses, we assign students the following problem (E_1):

In the space Oxyz, write the general equation of a plane (P) through point A(3, 2, -4)and

1. *parallel to straight lines:* (d$_1$) : $\begin{cases} x = 8 + t \\ y = 5 + 2t \\ z = 8 - t \end{cases}$ *and (d$_2$):* $\dfrac{x-3}{-7} = \dfrac{y-1}{2} = \dfrac{z-1}{3}$;

2. *parallel to two straight lines (D$_1$) and (D$_2$) which* $\vec{u} = (-3; -4; 6)$ *and* $\vec{v} = (\dfrac{1}{2}; \dfrac{2}{3}; -1)$ *are their direction vectors, respectively.*

Note: (d$_1$) and (d$_2$) are skew; (D$_1$) and (D$_2$) are parallel.

Participants

Subjects: 309 grade 12th students (academic year 2014 -2015) from two schools: The High school "Thực hành Sư phạm" (Can Tho University, Vietnam) and the High school "Nguyễn Thị Định" (Bến Tre province, Vietnam). (see Table 1).

Data collecting and analyzing: These participants were assigned the problem (E_1) to solve. After the students finished doing the above problem, we analyzed their solutions to the problem on basis of the concept "didactical contract".

Table 1: Students investigated

School	Class	The number of students
High school Thực hành Sư phạm (Can Tho University)	12A1, 12A2, 12B1, 12B2	144
High school Nguyễn Thị Định (Bến Tre province)	12T1, 12A2, 12A4, 12A7, 12A8	165
Total		**309**

RESULTS AND DISCUSSION

Strategies used to solve (E_1) by students were presented in Table 2.

Table 2: Students' strategies for solving (E_1)

Question of (E_1)	Strategy		The number of students	%
1.	S (right)		295	95.47
	No answer		14	4.53
	Total		**N=309**	**100%**
2.	Finding out : $\vec{n} = \left[\vec{u}; \vec{v}\right] = \vec{0}$	According to (S): *0x+0y+0z=0 is the equation of the plane (wrong)*	197	63.75
		Answering: there is the infinite number of planes (right)	17	5.50
		Answering: No any plane (wrong)	45	14.56
	Finding out : $\overrightarrow{u}$ and $\overrightarrow{v}$ are the same direction	Answering: there is the infinite number of planes (right)	7	2.27
		Answering: No any plane (wrong)	20	6.48
	Considering $\overrightarrow{u}$ as a normal vector of the plane (wrong)		6	1.94
	No answer		17	5.5
	Total		**N=309**	**100%**

For "question 1", because (d_1) and (d_2) are skew, 95.47 % students applied S to produce the right answer. For "question 2", (D_1) and (D_2) are parallel ($\overrightarrow{u}$ and $\overrightarrow{v}$ are the same direction), so 63.75% of students applied S to give the wrong answer because a, b, c in the general of a plane: ax + by + cz + d = 0 have to satisfy the condition: $a^2 + b^2 + c^2$ not equal to zero. The above results showed that the hypothesis H could be accepted.

In order to prevent the above errors of students when solving the type of task (E), in teaching process, the teacher should modify the strategy S as follows:

- Step 1: Find coordinates of $\overrightarrow{u}_1$ (direction vector of d_1) and $\overrightarrow{u}_2$ (direction vector of d_2)

  Choose vector pair ($\overrightarrow{u}_1$ $\overrightarrow{u}_2$), to be vector pair of direction of plane (P);

- Step 2: Find the coordinates of normal vector of plane (P): $\vec{n} = \left[\overrightarrow{u}_1; \overrightarrow{u}_2\right]$;

- Step 3:

  In the case of $\vec{n} \neq \vec{0}$: The equation of plane (P): ax + by + cz + d = 0 was determined due to the coordinates of $\vec{n}$ and point A.

  In the case of $\vec{n} = \vec{0}$: there is the infinite numbers of planes satisfing the requirements of the problem; therefore, the general equation of the plane is not determined.

CONCLUSION

"Didactical contract" is an action rule, which is implicit, of teachers and students. It indicates that students have habits to solve a problem based on a process that their teacher introduced before; therefore, in teaching situation, the contract is broken down, then they will make errors. From this study, we could consider the didactical contract in teaching as a tool for finding out the errors of students.

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